Whistler Waves

n = index of refraction

$$n = \frac{kc}{\omega}$$

light in vacuum n=1

light through glass n = 1.3

Index of refraction for a whistler wave

$$n^2 = 1 - \frac{\omega_{pe}^2}{(\omega + \omega_{ci})(\omega - \omega_{ce})}$$

frequencies:

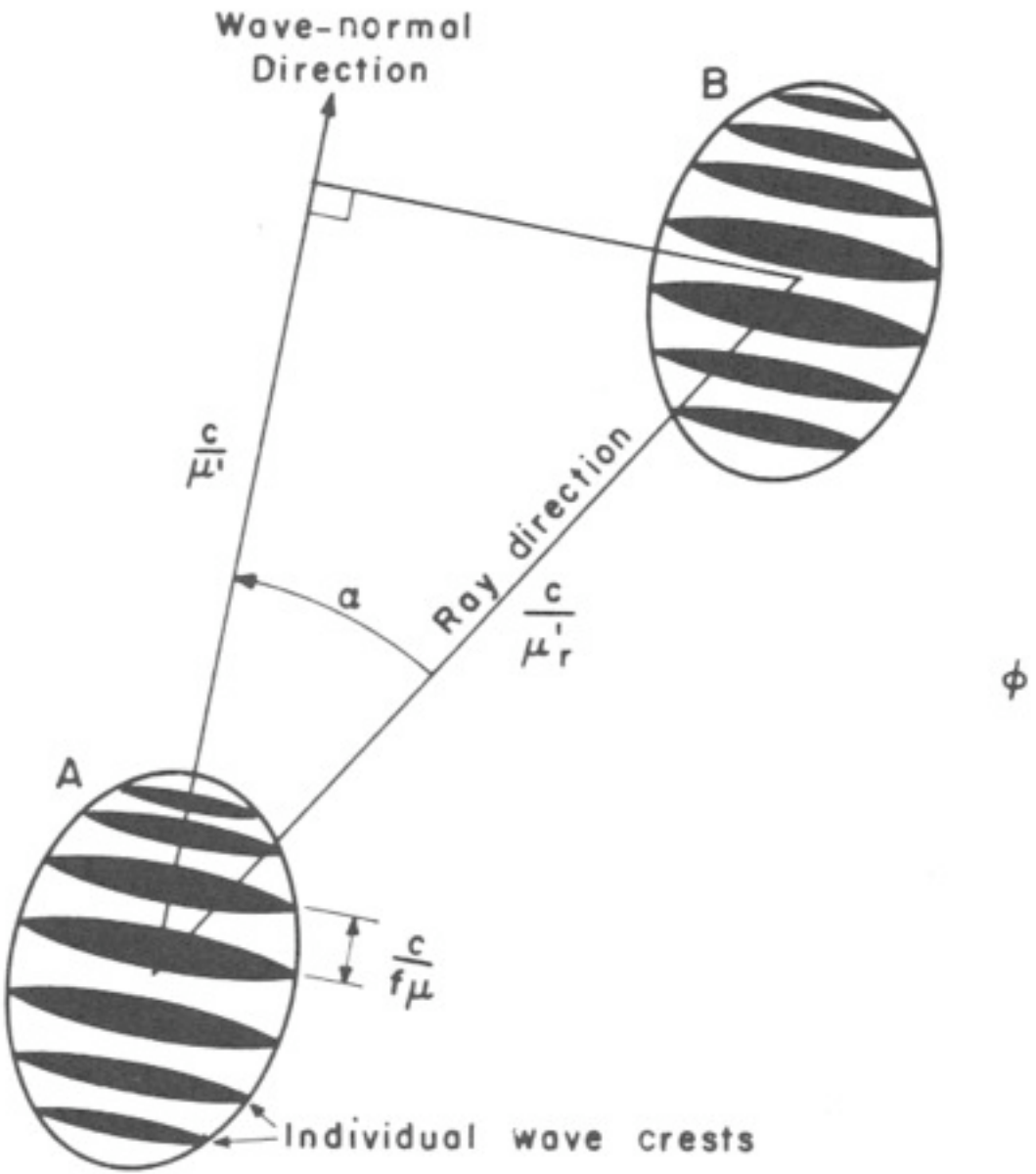
$$\omega = 2\pi f$$

$$f_{ce} = \frac{eB}{2\pi m_e} = 2.8 \times 10^6 B \quad \text{B(gauss)}$$

$$\mathrm{f}_{ci} = \frac{eB}{2\pi M_I} = 1.52 \times 10^3 \frac{B}{\mu} = 38B \text{ B(Gauss), Argon}$$

$$\mathrm{f}_{pe} = \frac{1}{2\pi}\sqrt{\frac{4\pi n e^2}{m_e}} = 8.98 \times 10^3 \sqrt{n} \quad \text{n density cm}^{-3}$$

Wave packet propagation



More propagation along B

$$n^2 = 1 - \frac{\omega_{pe}^2}{(\omega + \omega_{ci})(\omega - \omega_{ce})}$$

$$v_{phase} = \frac{\omega}{k} = \sqrt{\frac{\omega\omega_{ce}c^2}{\omega_{pe}^2}} \qquad \omega^2 = \frac{\omega\omega_{ce}c^2}{\omega_{pe}^2}k^2$$

$$\omega = \frac{\omega_{ce}c^2}{\omega_{pe}^2}k^2$$

$$d\omega = \frac{\omega_{ce}c^2}{\omega_{pe}^2}2kdk$$

$$v_{group} = \frac{\partial\omega}{\partial k} = \frac{2k\omega_{ce}c^2}{\omega_{pe}^2} \propto \omega^{\frac{1}{2}}$$

Ray Tracing
-- find out trajectory of a ray

Yuhou Wang

what if the wave is moving at an angle θ with respect to B?

Why ray tracing?

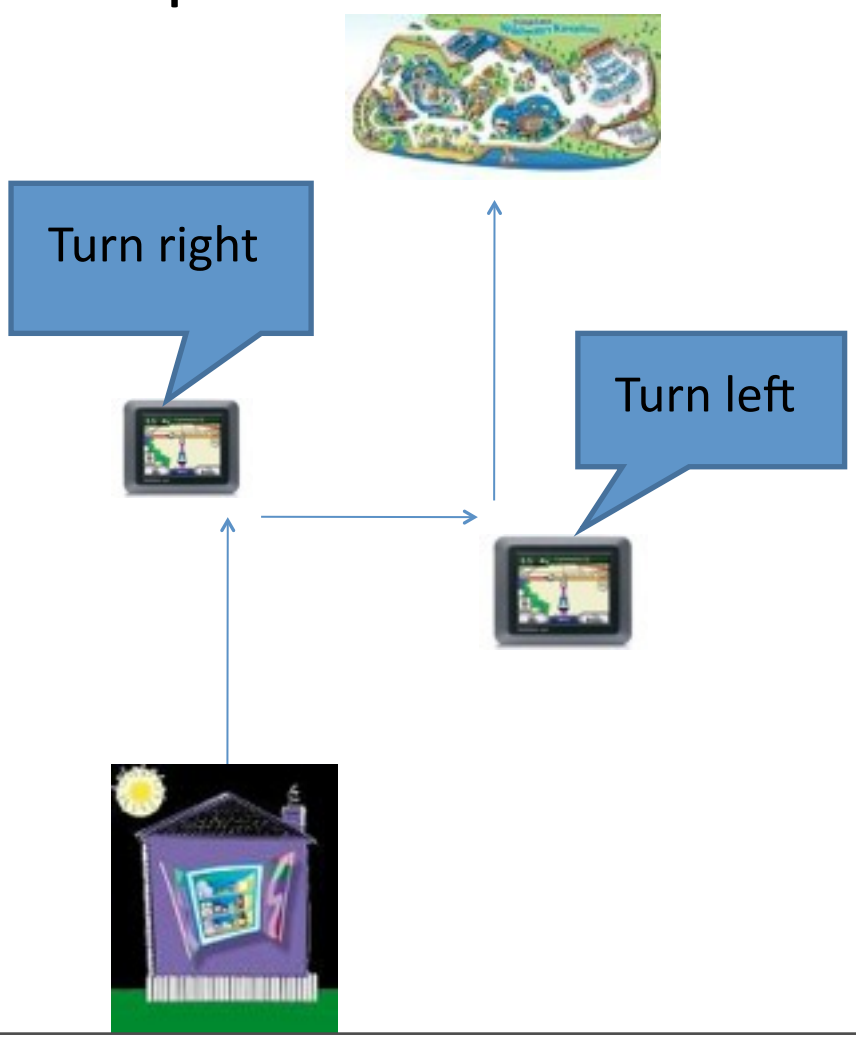
- Um…ray tracing?

  Like this?

What about waves in plasmas?

What's ray tracing

- Imagine you are going to a park …



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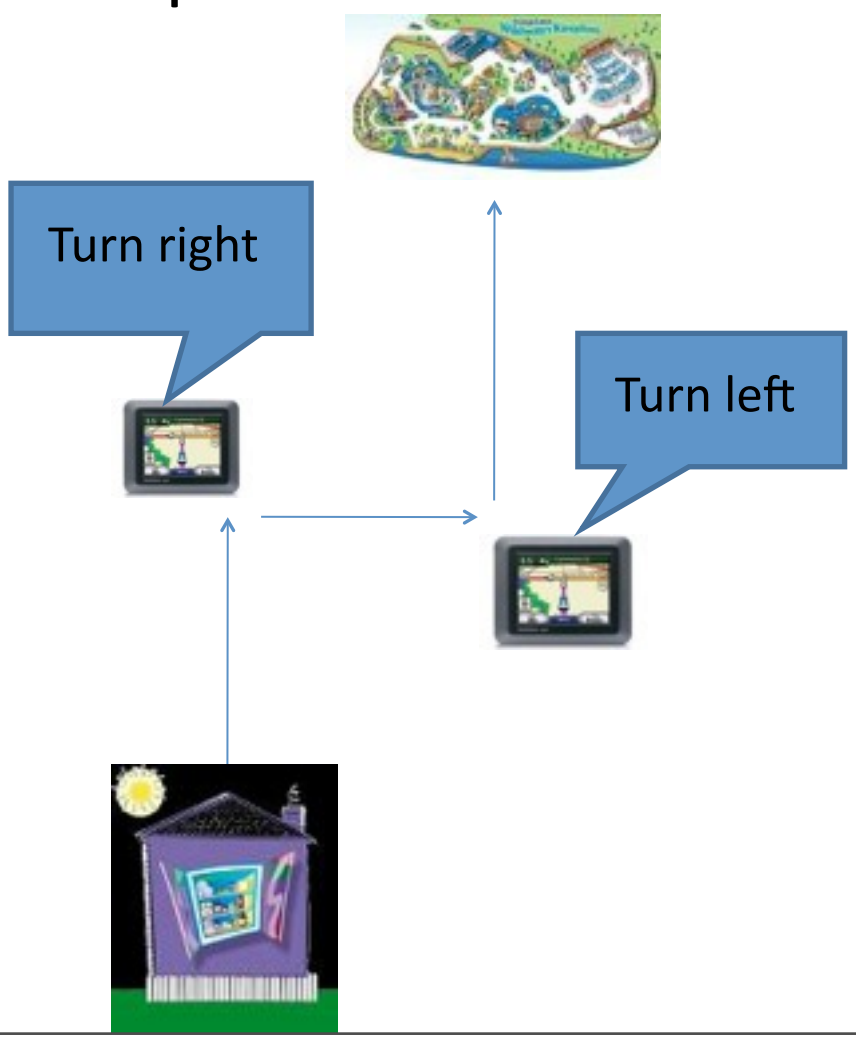

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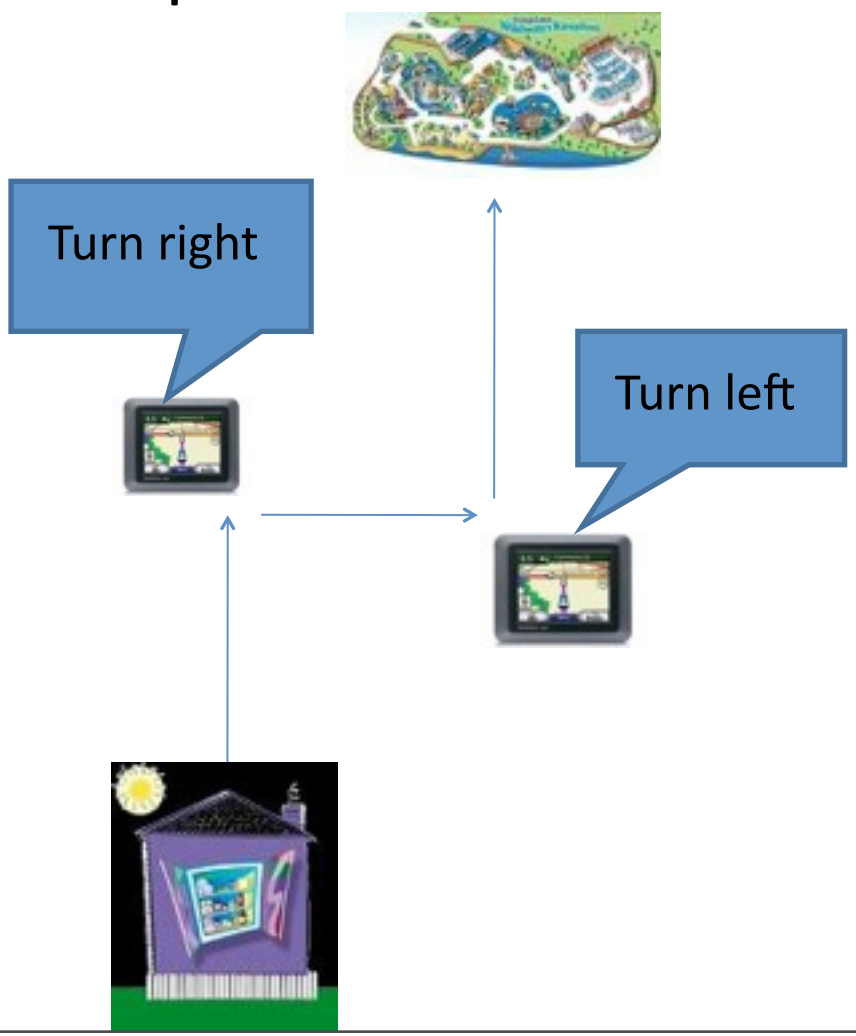

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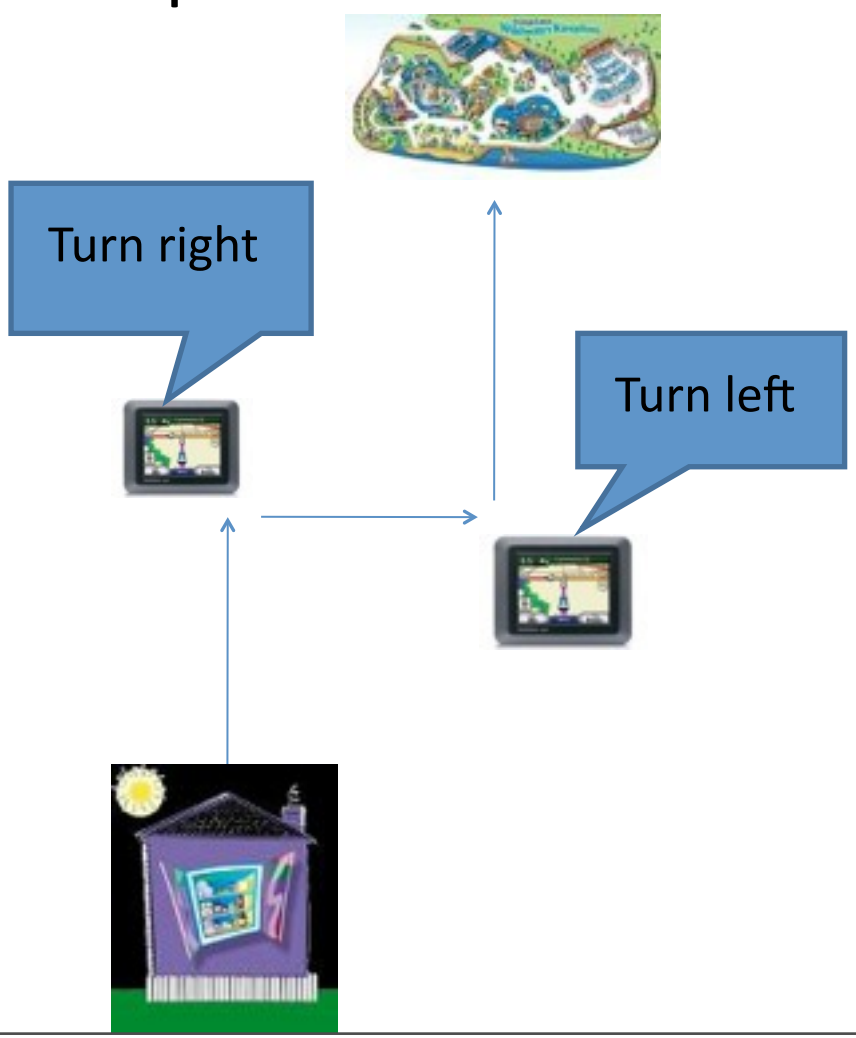

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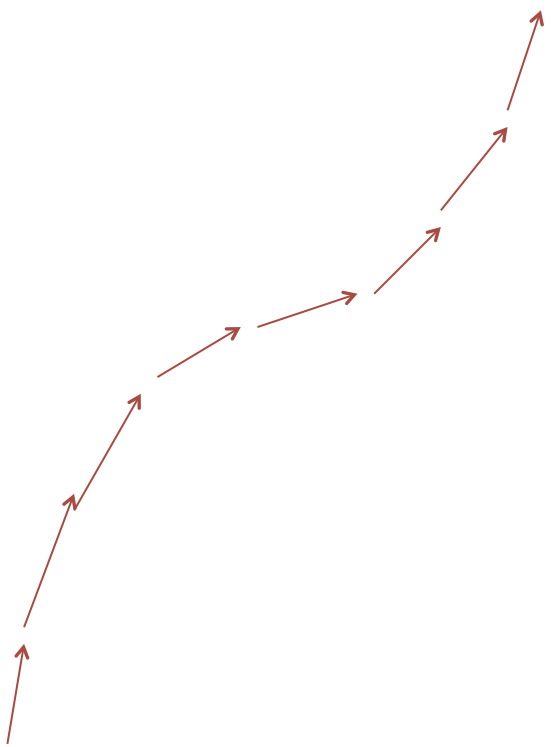

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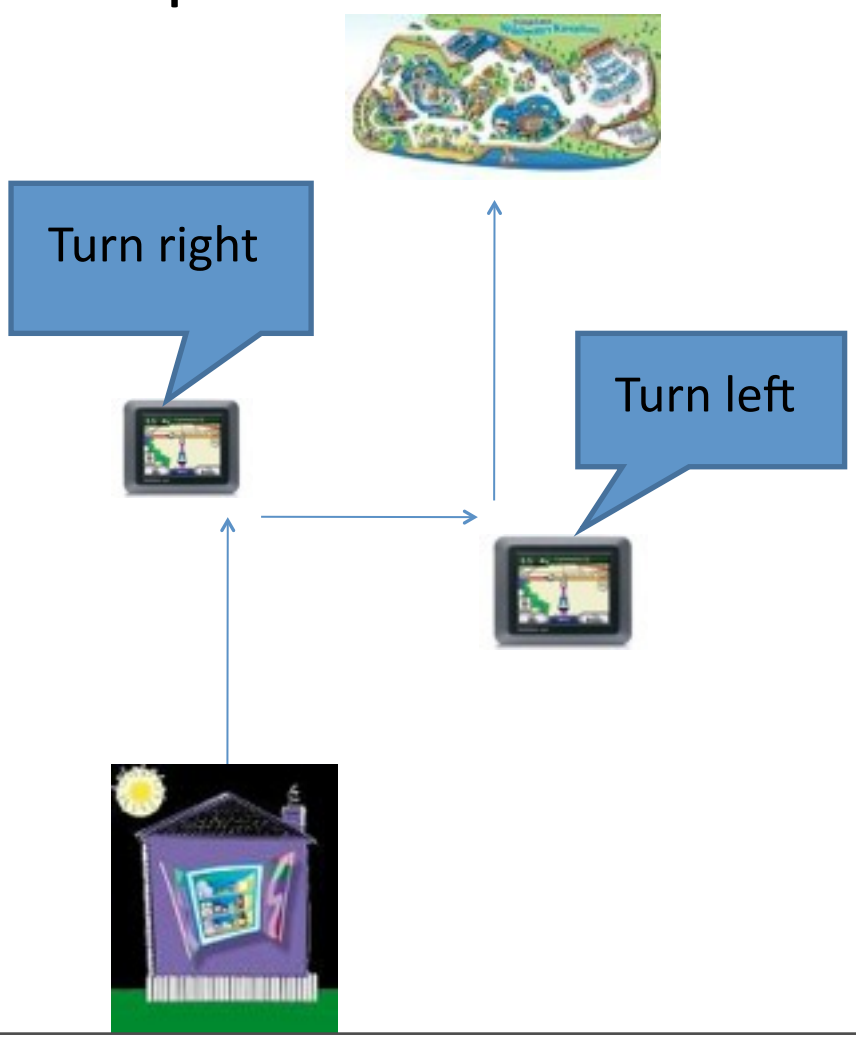

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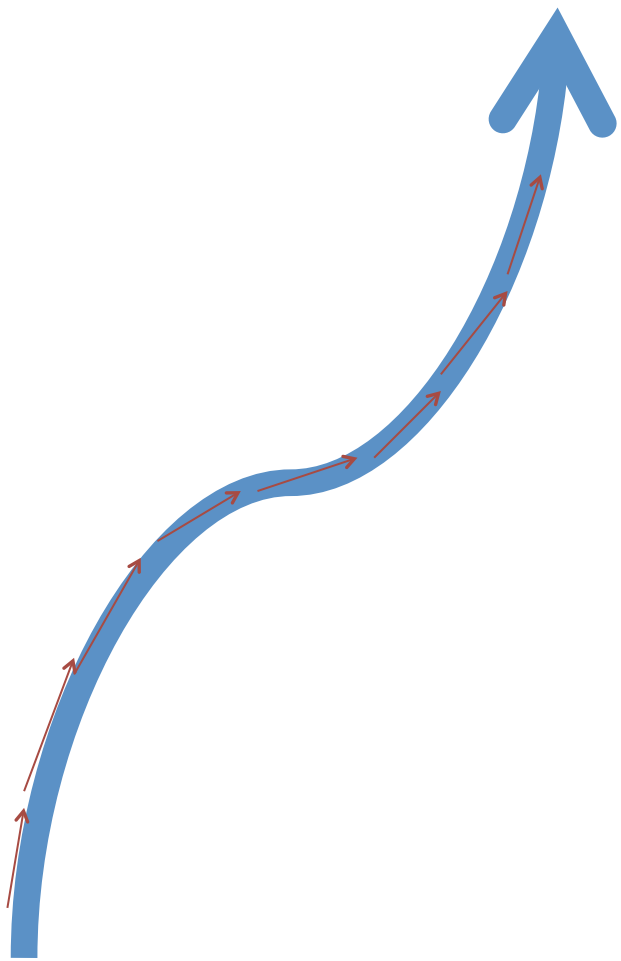
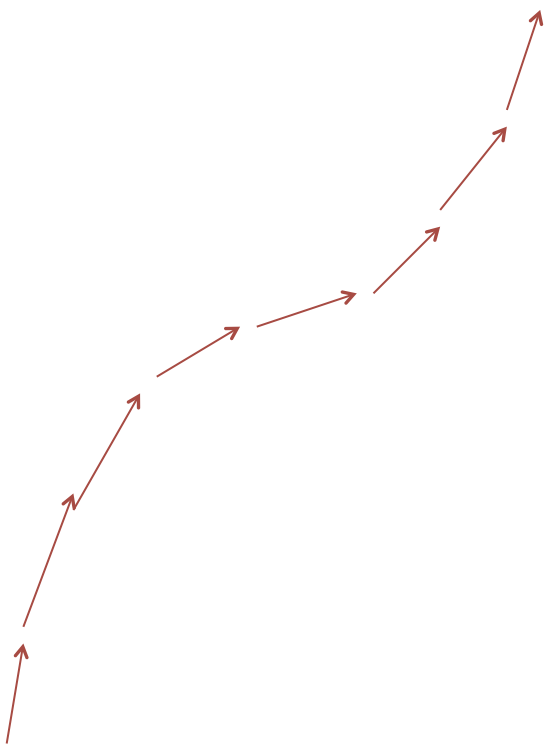


Coming up next …



- (How to decide the direction?)

  wave normal, group velocity

  refractive index surface

- (Back to our whistlers as an example)

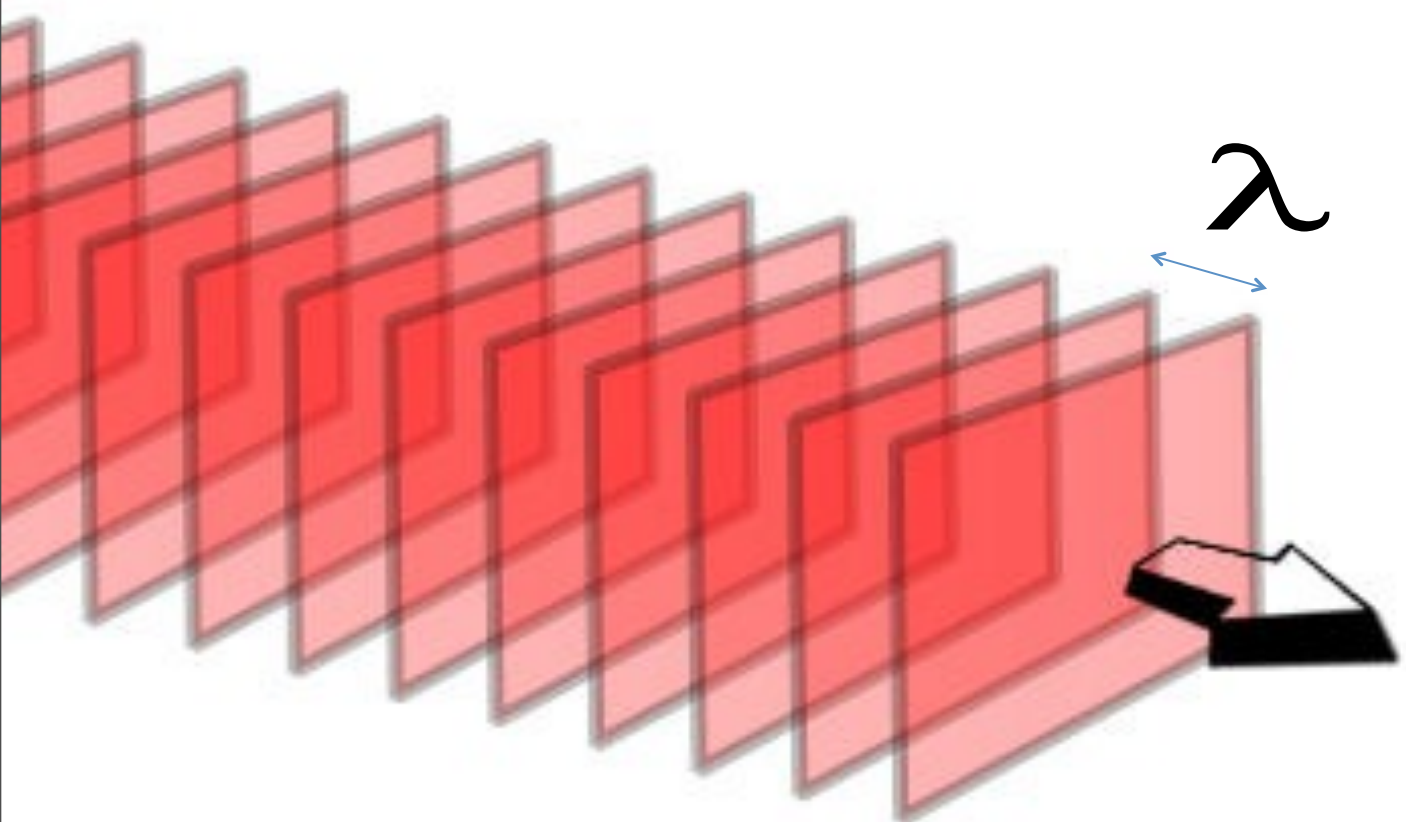
  refractive index of whistlers

  process of ray tracing

  Some interesting ray

Wave normal

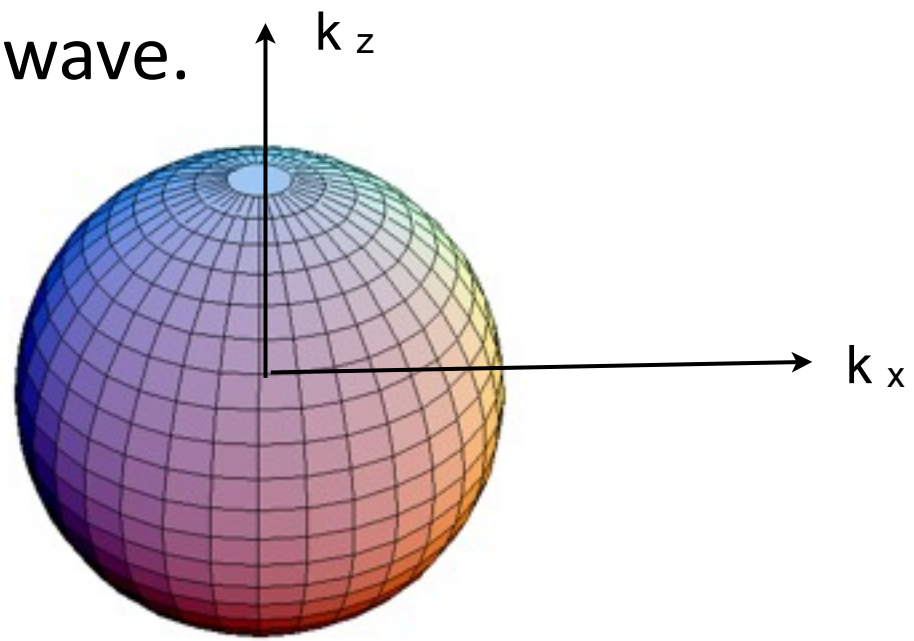
- Wave vector, k-vector



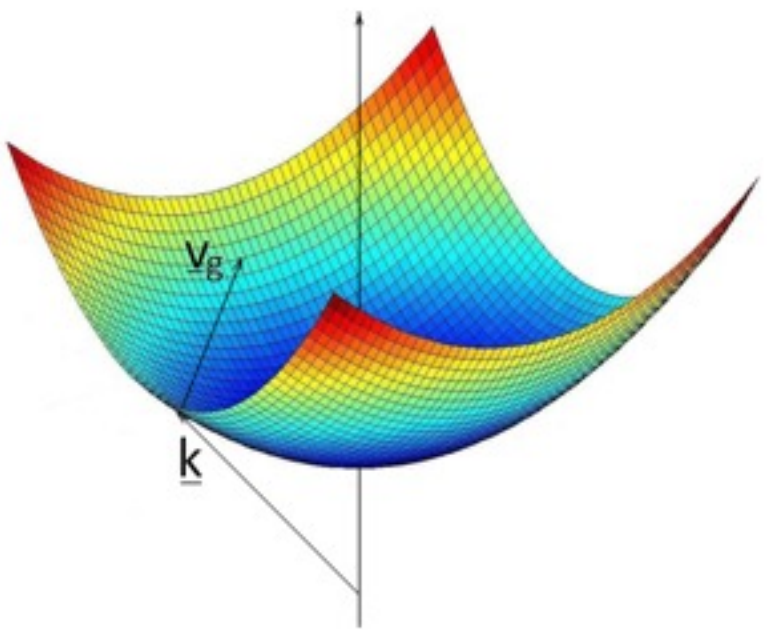
$$k = \frac{2\pi}{\lambda}.$$

refractive index surface

- Refractive index surface is a surface in k space, and it is a collection of all possible wave normal vectors for a certain wave.

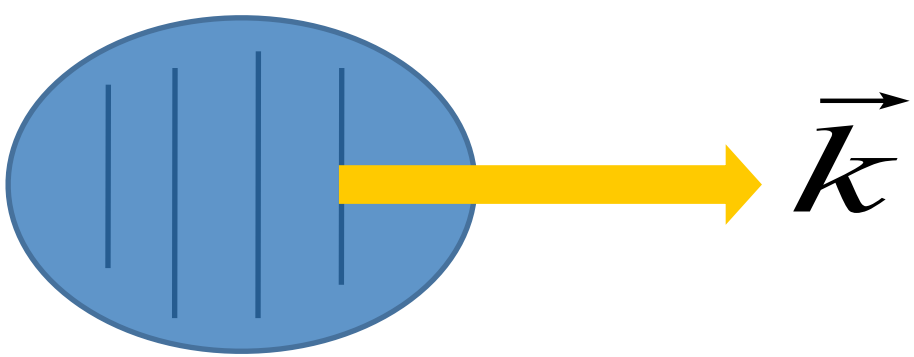


- Given a k-vector, the group velocity direction is perpendicular to refractive index surface at that point



Group velocity

- Defined as: $\vec{v}_g \equiv \frac{\partial \omega}{\partial \vec{k}}$
- Group velocity is the velocity at which energy propagates.



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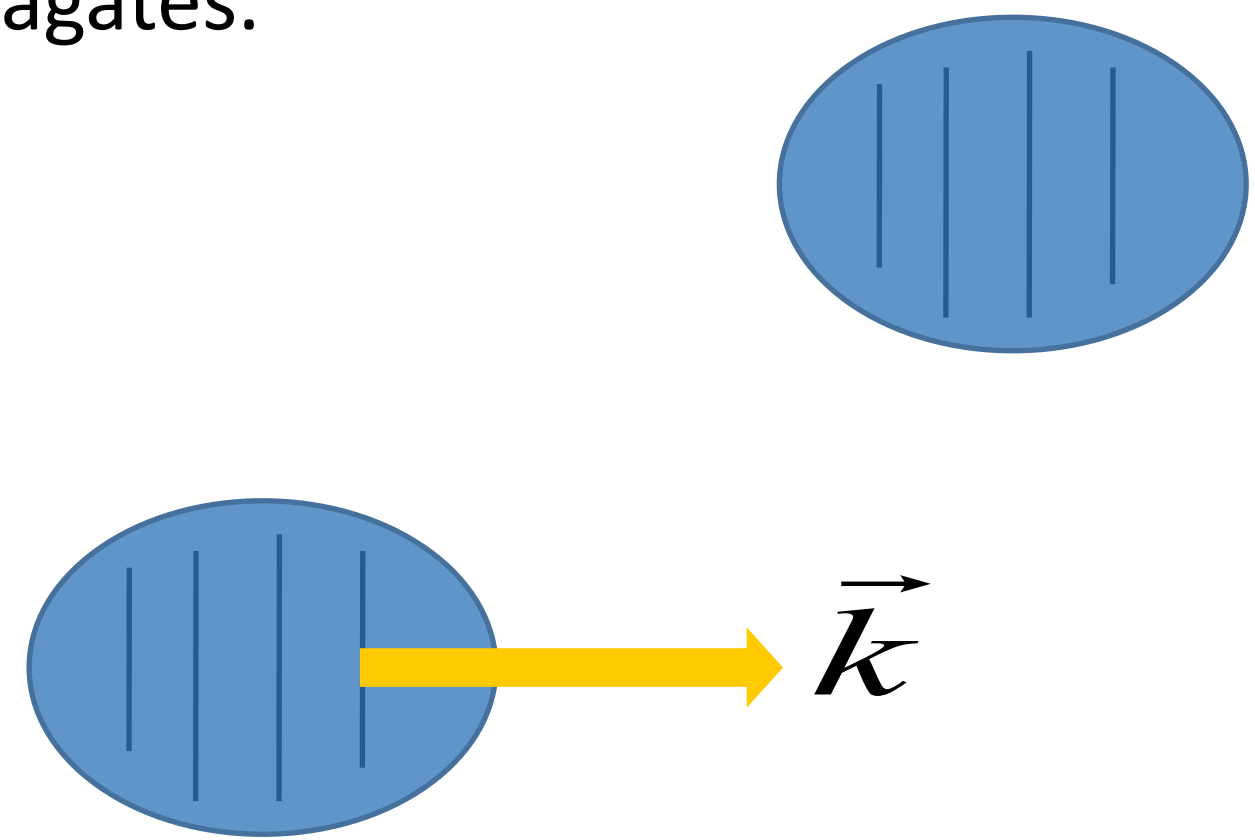

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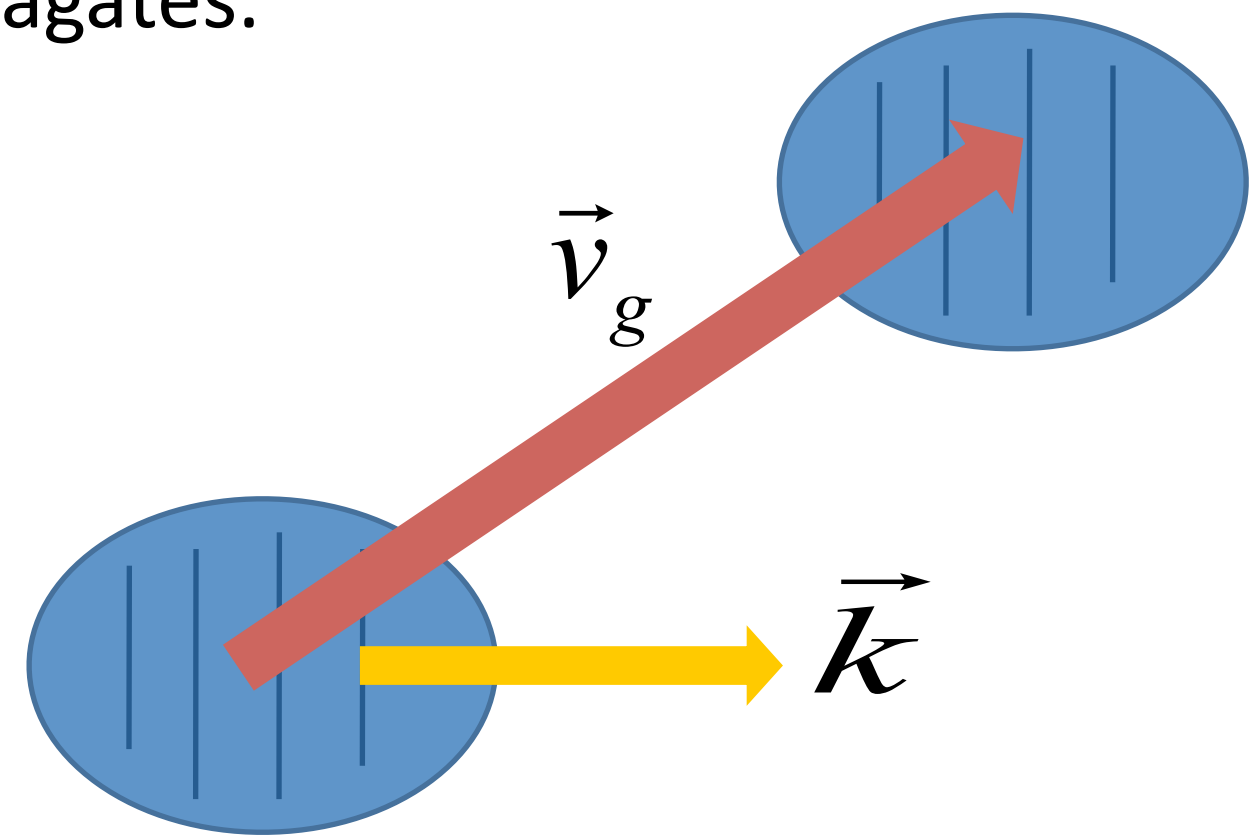
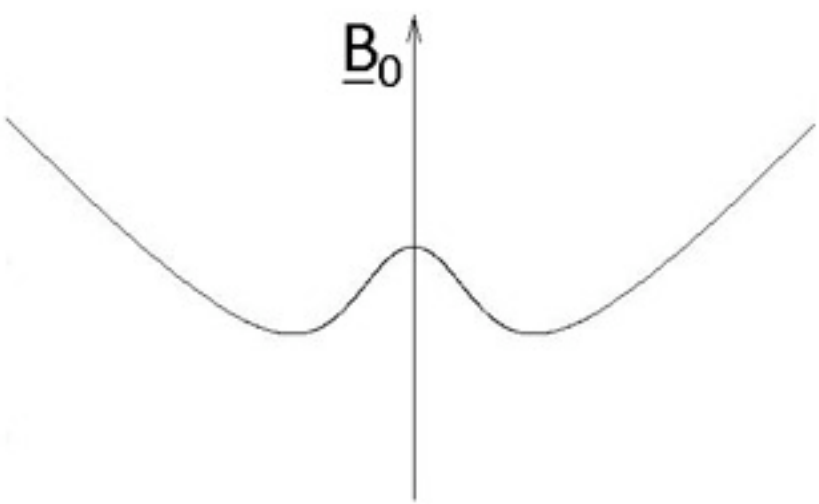


Refractive index surface – Whistler Wave

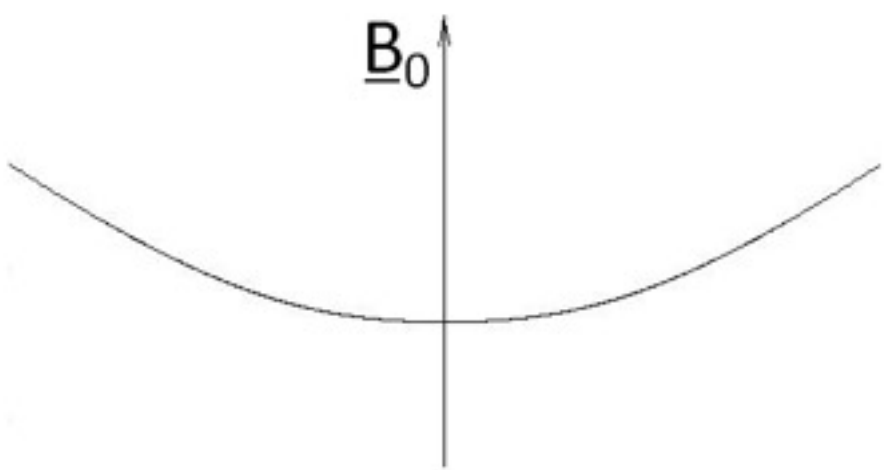
- Whistler wave refractive index:

$$n = (1 - \frac{\omega_{pe}^2 / \omega}{\omega - \omega_{ce}\cos\theta})^{1/2} = n(\omega, n_e, B_0, \theta)$$

$$0 < \frac{\omega}{\omega_{ce}} < \frac{1}{2}$$



$$\frac{1}{2} < \frac{\omega}{\omega_{ce}} < 1$$



Note at each density the refractive index surface changes...here is an example