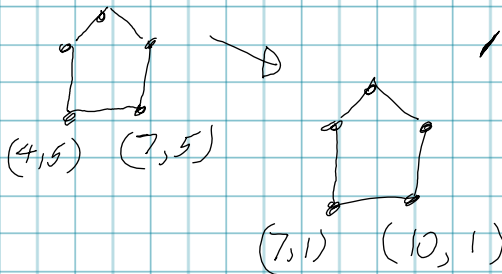


9/26/15 WALTER Lect

Pict of

How to translate and rotate object
mathematically?

Slide 2D Transformations -

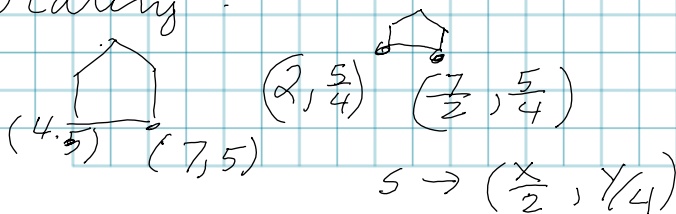


$$x' = x + dx \quad y' = y + dy$$

$$p = \begin{bmatrix} x \\ y \end{bmatrix} \quad p' = \begin{bmatrix} x' \\ y' \end{bmatrix} \quad t = \begin{bmatrix} dx \\ dy \end{bmatrix}$$

$$p' = p + t \quad \text{--- Translation}$$

Scaling



$$x' = S_x \cdot x \quad y' = S_y \cdot y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{array}{ccc} p' & = & S \cdot p \\ \uparrow & & \uparrow \quad \nwarrow \\ \text{column} & & \text{matrix} \quad \text{column} \\ \text{vector} & & \text{vector.} \end{array}$$

~~~~~  
Matrix  $\times$  scalar.

$$\vec{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\vec{A} \vec{A} = \vec{C}$$

$$\text{only if } c_{ij} = \sum_k a_{ik} b_{kj}$$

$$\alpha \vec{A} = \alpha a_{ij}$$

scalar

$$c_{ij} = a_{i1} b_{1j} + a_{i2} b_{2j} \dots$$

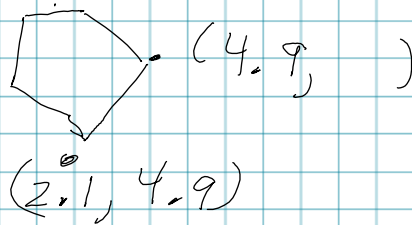
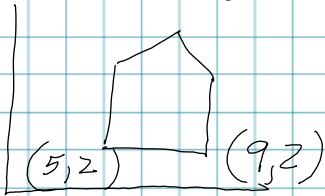
$$\vec{A} \vec{B} \neq \vec{B} \vec{A} \quad (AB)C = A(BC)$$

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

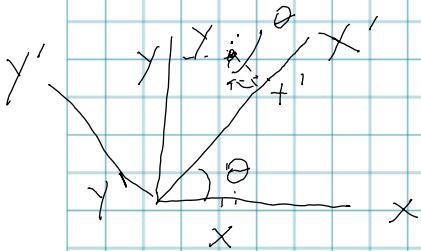
$$\vec{B} = \begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 7 & -1 \\ 23 & 6 \end{bmatrix}$$

Rotating an object in 2D



Distance from origin is constant.



$$x^2 + y^2 = R^2 = x'^2 + y'^2$$

$$x = x' \cos \theta - y' \sin \theta$$

$$y = x' \sin \theta + y' \cos \theta$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\vec{P}' = \vec{R} \cdot \vec{P}$$

$$P' = P + T$$

$$P' = \vec{J} \cdot P$$

$$P' = \vec{R} P$$

change 2D prob into 3D prob

$$P' = P + T \text{ old}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

by mult,  
conv change  
from + to X

$$P' = \vec{T} P \text{ Translation}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$P' = \vec{S} P$$

Rotation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$P' = \vec{R} P$$

all rotations must be done  
for objects centered at origin

$$T R T \begin{bmatrix} 1 & 0 & x_1 \\ 0 & 1 & y_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_1 \\ 0 & 1 & -y_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T(x_1, y_1) (R \theta) T(x_1, -y_1) = \begin{bmatrix} \cos \theta & -\sin \theta & x_1(1 - \cos \theta) + y_1 \sin \theta \\ \sin \theta & \cos \theta & y_1(1 - \cos \theta) - x_1 \sin \theta \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & x_1 \\ 0 & 1 & y_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_1 \\ 0 & 1 & -y_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\vec{X} \times \vec{J} = \vec{K} \Rightarrow \text{Right handed syst.}$$