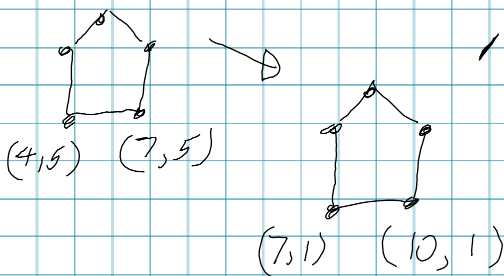
9/26/15 W After Lect

Pict of

How to translate and rotate object mathematically?

Slide 2D Transformations.

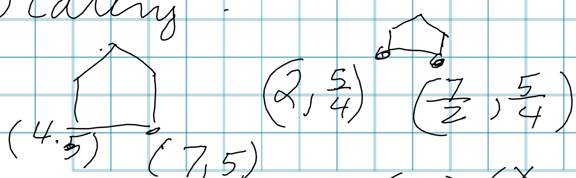


$x' = x + d_x \quad y' = y + d_y$

$p = \begin{bmatrix} x \\ y \end{bmatrix} \quad p' \begin{bmatrix} x' \\ y' \end{bmatrix} \quad T = \begin{bmatrix} d_x \\ d_x \end{bmatrix}$

$p' = p + T$ — Translation

Scaling.



$s \to \left(\frac{x}{2}, y/4\right)$

$x' = S_x \cdot x \qquad y' = S_y \cdot y$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$p' = \overleftrightarrow{S} \cdot p$$

column vector — matrix — column vector.

matrix x scalar.

$$\vec{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad \vec{A}\vec{A} = \vec{C}$$

only if $c_{ij} = \sum_k a_{ik} b_{kj}$

$$\alpha \vec{A} = \alpha a_{ij}$$

scalar

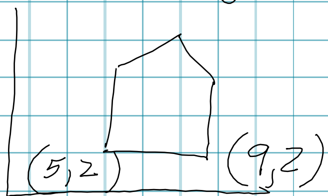
$$c_{ij} = a_{i1} b_{1j} + a_{i2} b_{2j} \ldots$$

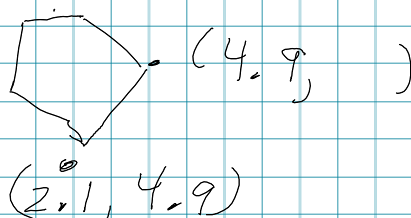
$$\vec{A}\vec{B} \neq \vec{B}\vec{A} \qquad (AB)C = A(BC)$$

$A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ $\qquad \vec{B} = \begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix}$

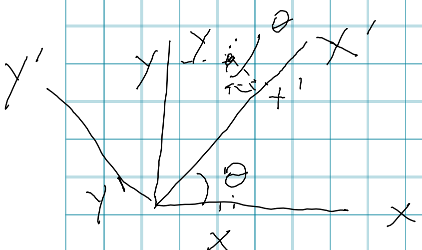
$AB = \begin{bmatrix} 7 & -1 \\ 23 & 6 \end{bmatrix}$

Rotating an object in 2D





Distance from origin is constant.



$x^2 + y^2 = R^2 = x'^2 + y'^2$

$x = x'\cos\theta \quad y'\sin\theta$

$y = x'\sin\theta + y'\cos\theta$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\vec{p}\,' = \vec{R}_\theta \vec{p}$$

$$p' = p + T$$

$$p' = \vec{S} \cdot p$$

$$p' = \vec{R} p$$

change 2D prob into 3D prob

$p' = p + T$ old

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

by multi, can change from + to ×

$$p' = \vec{T} p$$

Translation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$P' = \vec{S}\vec{P}$$

<u>Rotation</u>

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$P' = \vec{R}\vec{P}$$

all rotation must be done for objects <u>centered</u> at origin

$$TRT \begin{bmatrix} 1 & 0 & x_1 \\ 0 & 1 & y_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_1 \\ 0 & 1 & -y_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T(x_1,y_1)(R\theta)T(-x_1,-y_1) = \begin{bmatrix} \cos\theta & -\sin\theta & x_1(1-\cos\theta)+y_1\sin\theta \\ \sin\theta & \cos\theta & y_1(1-\cos\theta)-x_1\sin\theta \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & x_1 \\ 0 & 1 & y_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_1 \\ 0 & 1 & -y_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$\vec{X} \times \vec{J} = \vec{K}$ Right handed syst.