

9/20/16 Walter Whistler Waves.

Real Force Egn.

$$m \left( \frac{d\vec{v}}{dt} + (\vec{v} \cdot \nabla) \vec{v} \right) = -\nabla \rho + q(\vec{E} + \vec{v} \times \vec{B})$$

small  
non linear  
neglect

$$m v \vec{v}$$
$$\frac{d}{dt} = -i\omega$$
$$\nabla \times \rightarrow i k \times$$

$$\vec{B} = B_0 \hat{z} + B_{\perp} \hat{e}_\perp$$

Maxwell's Egn's -

①  $\nabla \cdot \vec{B} = 0$  no magnetic ~~field~~ monopoles

②  $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$  Gauss/Coulomb Law

③  $\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{d\vec{E}}{dt}$  Ampere/Max  
LAW

④  $\nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$  Faraday's Law

$$\vec{J} = -en\vec{v}$$

current -

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot \vec{n} dA$$

$$B_1 = B_0 e^{i k_x x} e^{-i \omega t} e^{i k_y y}$$

$$\nabla \times \vec{E} = -\frac{d}{dt} \vec{B}$$

$$\textcircled{4} \quad i \vec{k} \times \vec{E} = i \omega \vec{B}_1$$

$$\textcircled{3} \quad i k_x B_1 = -n e \rho_0 \vec{v}_1 - i \mu_0 \epsilon_0 \omega \vec{E}_1$$

Assume  $B(\vec{r}, t) = B_0 + B_1(\vec{r}, t) \quad \vec{B}_1 \ll \vec{B}_0$   
n

Calc P  $\rho = 0 \quad \nabla P = 0$

E

$$\vec{k} \times \vec{E} = \omega \vec{B}_w$$

$$i k_x B_w = -\mu_0 (e n \vec{v}) - i \omega \mu_0 \epsilon_0 \vec{E}$$

$$-i \omega m \vec{v} = -e (\vec{E} + \vec{v} \times (\vec{B}_0)) - m \gamma \vec{v} \quad \underline{F = ma}$$

$$\textcircled{a} \quad \frac{1}{\omega} \vec{k} \times E = \vec{B}_\omega$$

Ampere Law.

$$\textcircled{b} \quad i \vec{k} \times \left( \frac{1}{\omega} \vec{k} \times E \right) = -\mu_0 (en \vec{v}) - \omega \mu_0 \epsilon_0 \vec{E}$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B} (\vec{A} \cdot \vec{C}) - \vec{C} (\vec{A} \cdot \vec{B})$$

$$\vec{k} \times (\vec{k} \times E) = \vec{k} (\vec{k} \cdot E) - k^2 E$$

$$\textcircled{c} \quad i \vec{k} (\vec{k} \cdot E) - i k^2 E = -\mu_0 (en \vec{v}) - \omega \mu_0 \epsilon_0 E$$

$$\nabla \times B = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{dE}{dt} \quad c^2 = \frac{1}{\mu_0 \epsilon_0}$$

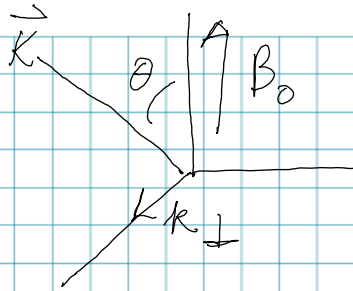
$$\left( \frac{i \omega en_0 \vec{v}}{\epsilon_0 c^2} \right) = \frac{\omega^2}{c^2} E + \vec{k} (\vec{k} \cdot E) - k^2 E$$

$$-i \omega m \vec{v} = -e (\vec{E} + \vec{v} \times (\vec{B}_0)) - m v \vec{v}$$

$$i \omega m \vec{v} = i e (\vec{E} + \vec{v} \times (\vec{B}_0)) + i m v \vec{v}$$

$$\left( 1 + \frac{i v}{\omega} \right) \vec{v} + \frac{i e}{m \omega} (\vec{v} \times \vec{B}_0) = \frac{-i e}{m \omega} E$$

$$\vec{B}_0 = B_0 \hat{k}$$



$$\vec{k} = k \sin \theta \hat{i} + k \cos \theta \hat{k}$$

$$\frac{i \omega \epsilon_0 n_0 \vec{V}}{\epsilon_0 c^2} = \frac{\omega^2}{c^2} \vec{E} + \vec{k} (\vec{k} \cdot \vec{E}) - k^2 \vec{E}$$

$$\vec{k} (\vec{k} \cdot \vec{E}) - k^2 \vec{E} =$$

$$(k \sin \theta \hat{i} + k \cos \theta \hat{k}) \cdot (k \sin \theta \hat{i} + k \cos \theta \hat{k}) \vec{E}$$

$$(E_x \hat{i} + E_y \hat{j} + E_z \hat{k}) \cdot$$

$$k^2 (E_x \hat{i} + E_y \hat{j} + E_z \hat{k})$$

$$\vec{k} (\vec{k} \cdot \vec{E}) - k^2 \vec{E} = (k^2 \sin^2 \theta E_x \hat{i} +$$

$$k^2 \sin \theta \cos \theta E_z \hat{i} - k^2 E_x \hat{i})$$

$$- k^2 E_y \hat{j}$$

$$- k^2 (E_z + E_x \cos \theta \sin \theta)$$

$$\left( \frac{\mu_0 \epsilon_0 \dot{V}_x}{\epsilon_0 c^2} \right) = \frac{\omega^2}{c^2} E_x + \frac{\mu_0^2}{c^2} \cos \theta (\sin \theta E_z - \cos \theta E_x)$$

$$\frac{\mu_0 \epsilon_0 \dot{V}_x}{\epsilon_0 \omega} = E_x \left[ 1 - \frac{\mu_0^2 c^2}{\omega^2} \cos^2 \theta \right] + \frac{\mu_0^2 c^2}{\omega^2}$$

$$\left( \frac{\mu_0 \epsilon_0 \dot{V}_z}{\epsilon_0 \omega} \right) = E_z + \frac{\mu_0^2 c^2}{\omega^2} (\sin \theta E_x - \dots)$$

$$\mathbf{V} \times \mathbf{B} = B_0 (V_x \hat{i} - V_y \hat{j})$$

$$\left(1 + \frac{iV}{\omega}\right) v_x + \frac{ieB_0}{\omega m} v_y = \frac{ie}{m\omega} E_x$$

$$\left(1 + \frac{iV}{\omega}\right) v_y - \frac{i\omega ce}{\omega} v_x = \frac{ie}{m\omega} E_y$$

$$v_x + \frac{iV}{\omega} v_y = \frac{ie}{m\omega} E_x$$

~~Project 2~~ Whistler wave. 2/27/16  
Walter.

index of refraction  $n = \frac{kc}{\omega}$

$$\begin{pmatrix} 1 - n^2 \cos^2 \theta & 0 & n^2 \sin \theta \cos \theta \\ 0 & (1 - n^2) & 0 \\ n^2 \sin \theta \cos \theta & 0 & (1 - n^2 \sin^2 \theta) \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} =$$

$$\begin{pmatrix} \frac{ien}{\epsilon_0 \omega} \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$$