9/20/16 Walter Whistler Waves.

Real Force Eqn.

$$m\left(\frac{\partial \vec{V}}{\partial t} + (\vec{V}\cdot\nabla)\vec{V}\right) = -\nabla p + q(\vec{E} + \vec{V}\times\vec{B}) - m\nu\vec{V}$$

small
nonlinear
neglect

$\frac{\partial}{\partial t} = -i\omega$

$\nabla\times \to ik\times$

$\vec{B} = \vec{B}_{0z} + B_w$

Maxwell's Eqns -

1. $\nabla\cdot B = 0$ no magnetic ~~field~~ monopoles
2. $\nabla\cdot E = \frac{\rho}{\varepsilon_0}$ Gauss/Coulomb Law
3. $\nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$ Ampere/Max LAW
4. $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ Faraday's Law

$J = -en\vec{V}$
current.

$$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int \vec{B} \cdot \hat{n} \, dA$$

$$B_1 = B_0 e^{i k_x x} e^{-i\omega t} e^{i k_y y}$$

$$\nabla \times E = -\frac{\partial}{\partial t} \vec{B}$$

(4) $i\vec{K} \times \vec{E} = i\omega \vec{B}_1$

(3) $i K_x B_1 = -ne\mu_0 \vec{V}_1 - i\mu_0 \epsilon_0 \omega \vec{E}_1$

Assume $B(\vec{r}, t) = B_0 + B_1(\vec{r}, t)$ $\quad \vec{B}_1 \ll \vec{B}_0$

n

Cold p $\quad p = 0 \quad \nabla P = 0$

E

$$\vec{K} \times \vec{E} = \omega B_w$$

$$i K \times B_w = -\mu_0 (en\vec{v}) - i\omega \mu_0 \epsilon_0 \vec{E}$$

$$-i\omega m \vec{v} = -e(\vec{E} + \vec{v} \times (\vec{B}_0)) - m\nu \vec{v} \quad F = ma$$

(a) $\frac{i}{\omega}\vec{K}\times\vec{E}=\vec{B}_\omega$

Amps Law

(b) $i\vec{K}\times\left(\frac{i}{\omega}K\times\vec{E}\right)=-\mu_0(en\vec{v})-\omega\mu_0\varepsilon_0\vec{E}_0$

$$\vec{A}\times(\vec{B}\times\vec{C})=\vec{B}(\vec{A}\cdot\vec{C})-\vec{C}(\vec{A}\cdot\vec{B})$$

$$\vec{K}\times(\vec{K}\times\vec{E})=\vec{K}(\vec{K}\cdot\vec{E})-K^2\vec{E}$$

(b) $i\vec{K}(\vec{K}\cdot\vec{E})-iK^2\vec{E}=-\mu_0(\omega en\vec{v})-i\omega^2\mu_0\varepsilon_0\vec{E}$

$$\nabla\times B=\mu_0\vec{j}+\mu_0\varepsilon_0\frac{\partial E}{\partial t}\qquad c^2=\frac{1}{\mu_0\varepsilon_0}$$

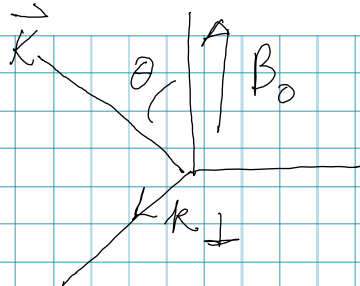
$$\left(\frac{i\omega en_0\vec{v}}{\varepsilon_0 c^2}\right)=\frac{\omega^2}{c^2}\vec{E}+\vec{k}(\vec{K}\cdot\vec{E})-k^2\vec{E}$$

$$-i\omega m\vec{v}=-e(\vec{E}+\vec{v}\times(\vec{B}_0))-m\nu\vec{v}$$

$$\omega m\vec{v}=ie(\vec{E}+v\times(\vec{B}_0))+im\nu\vec{v}$$

$$\left(1+\frac{i\nu}{\omega}\right)\vec{v}+\frac{ie}{m\omega}(\vec{v}\times\vec{B}_0)=\frac{-ie}{m\omega}\vec{E}$$

$$\vec{B}_0 = B_0\hat{k}$$



$$\vec{k} = k\sin\theta\,\hat{i} + k\cos\theta\,\hat{k}$$

$$\frac{i\omega e n_0 \vec{V}}{\varepsilon_0 c^2} = \frac{\omega^2}{c^2}\vec{E} + \vec{k}(\vec{k}\cdot\vec{E}) - k^2\vec{E}$$

$$\vec{k}(\vec{k}\cdot\vec{E}) - k^2\vec{E} =$$

$$(k\sin\theta\,\hat{i} + k\cos\theta\,\hat{k})\{(k\sin\theta\,\hat{i} + k\cos\theta\,\hat{k})\cdot$$

$$(E_x\hat{i} + E_y\hat{j} + E_z\hat{k}) -$$

$$-k^2(E_x\hat{i} + E_y\hat{j} + E_z\hat{k})$$

$$\vec{k}(\vec{k}\cdot\vec{E}) - k^2\vec{E} = (k^2\sin^2\theta\,E_x\hat{i} +$$

$$k^2\sin\theta\cos\theta\,E_z\hat{i} - k^2E_x\hat{i})$$

$$-k^2E_y\hat{j}$$

$$-k^2(E_z + E_x\cos\theta\sin\theta$$

$$\left(\frac{i\omega e n_0}{\varepsilon_0 c^2}\right) = \frac{\omega^2}{c^2} E_x + k^2 \cos\theta \left(\sin\theta E_z - \cos\theta E_x\right)$$

$$\frac{i e n_0 V_x}{\varepsilon_0 \omega} = E_x \left[1 - \frac{k^2 c^2}{\omega^2} \cos^2\theta\right] + \frac{k^2 c^2}{\omega^2}$$

$$\left(\frac{i e n_0 V_z}{\varepsilon_0 \omega}\right) = E_z + \frac{k^2 c^2}{\omega^2}\left(\sin\theta E_x - \ldots\right.$$

$$V \times B = B_0 (V_y \hat{i} - V_x \hat{j})$$

$$\left(1+\frac{i\nu}{\omega}\right)V_x+\frac{ieB_0}{\omega m}V_y=\frac{ie}{m\omega}E_x$$

$$\left(1+\frac{i\nu}{\omega}\right)V_y-\frac{i\omega_{ce}}{\omega}V_x=\frac{ie}{m}E_y$$

$$UV_x + \cancel{\ } iYV_y = ieE_x$$

Lect 2 Whistler Wave. 2/27/16

Walter.

index of refraction $n = \frac{Kc}{\omega}$

$$\begin{pmatrix} 1-n^2\cos^2\theta & 0 & n^2\sin\theta\cos\theta \\ 0 & (1-n^2) & 0 \\ n^2\sin\theta\cos\theta & 0 & (1-n^2\sin^2\theta) \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} =$$

$$\left(\frac{ien}{\varepsilon_0\omega}\right)\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$$