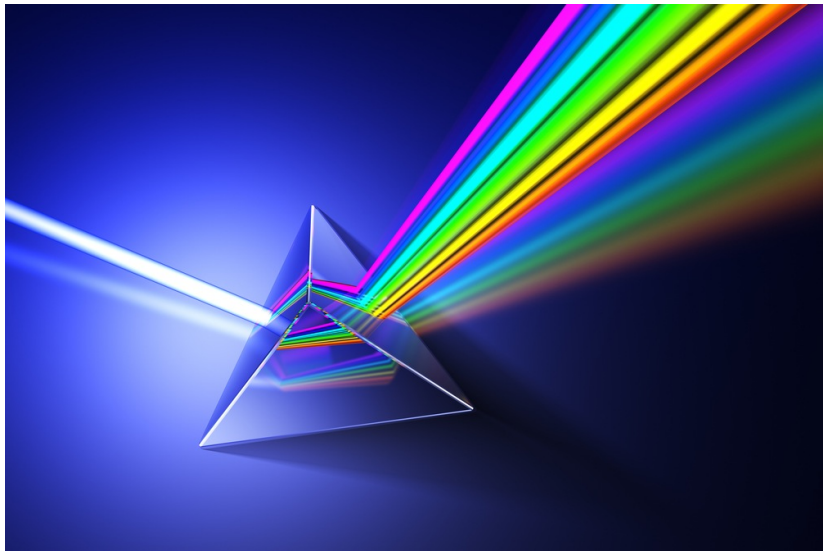
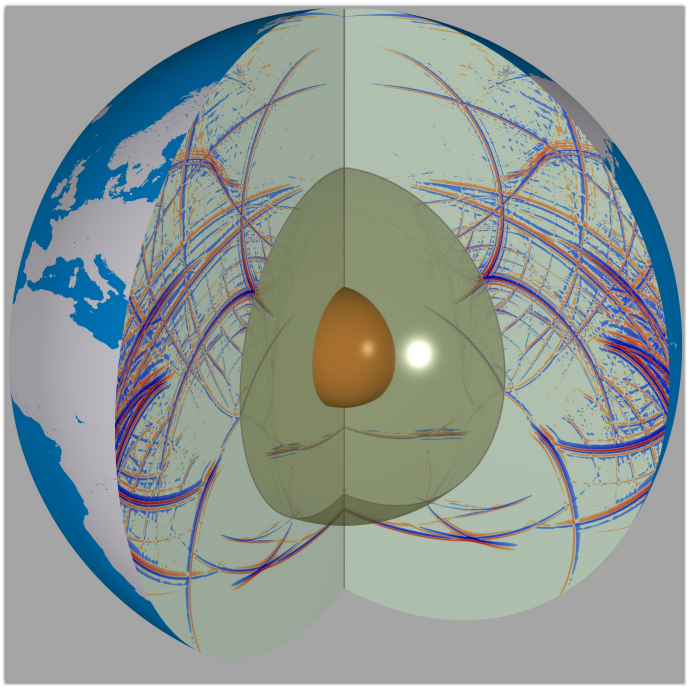
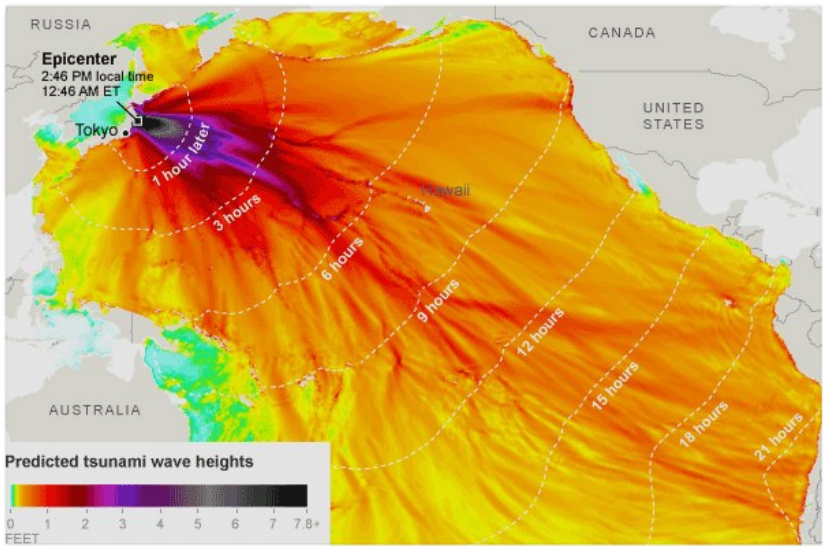
Mode conversion
between
X wave - Electron Bernstein Wave

Mode conversions

- when propagation of energy changes form

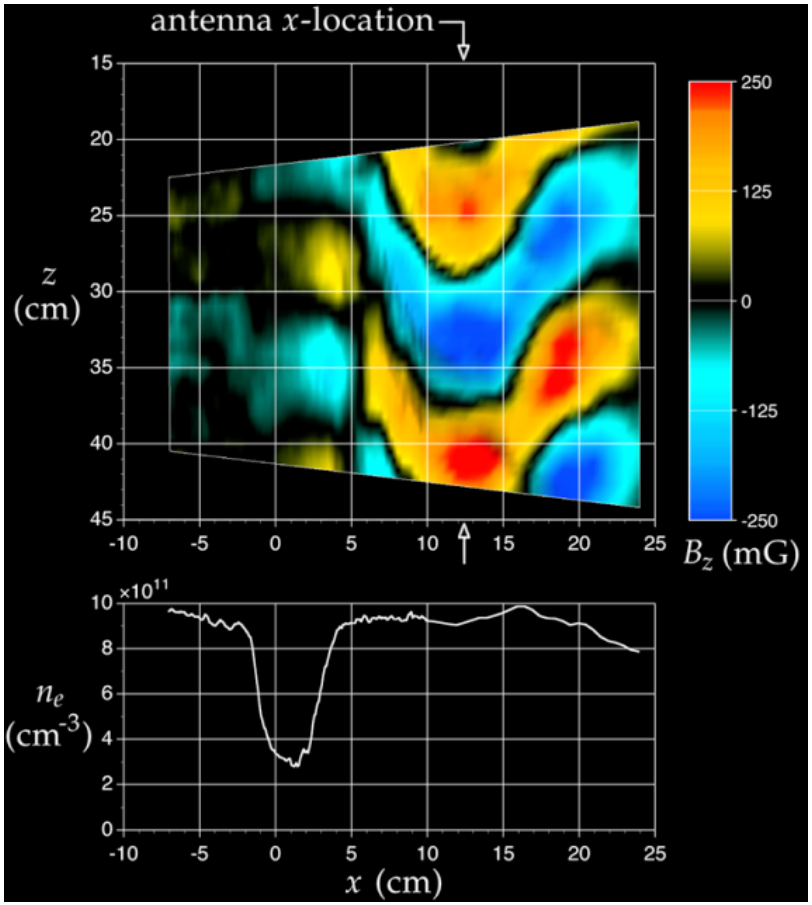




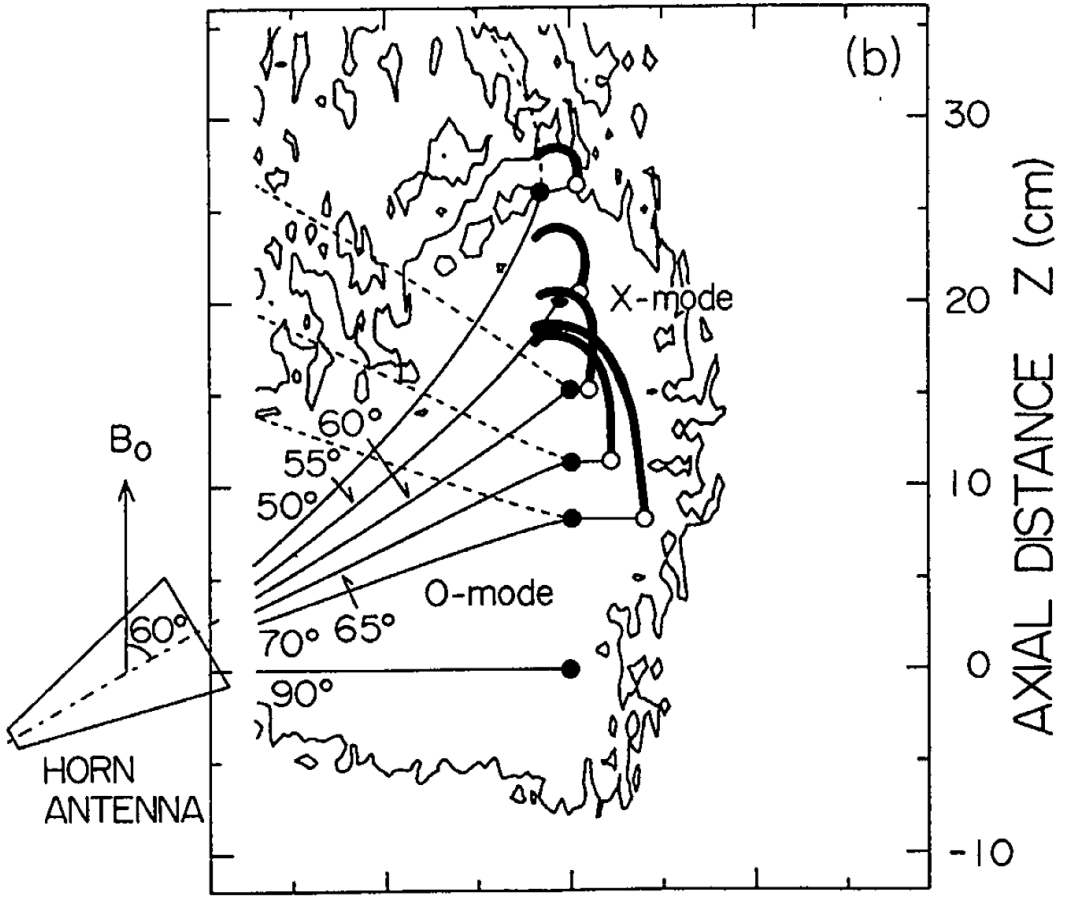


- light waves
- seismic waves
- water waves
- …

… more examples of mode conversions between plasma waves



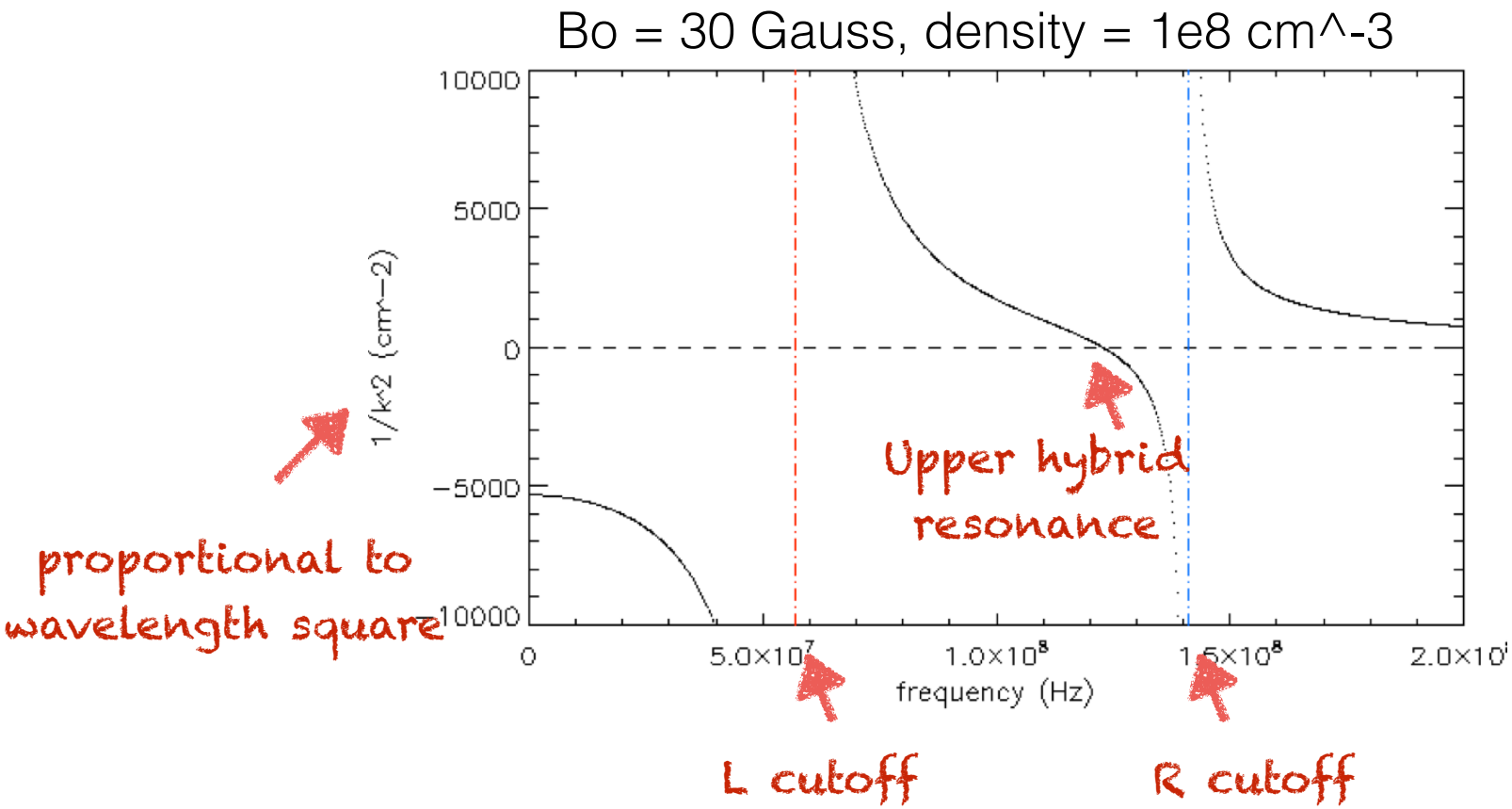
Bamber, J. F., J. E. Maggs, and W. Gekelman. "Whistler wave interaction with a density striation: A laboratory investigation of an auroral process." Journal of Geophysical Research: Space Physics (1978–2012) 100.A12 (1995): 23795-23810.



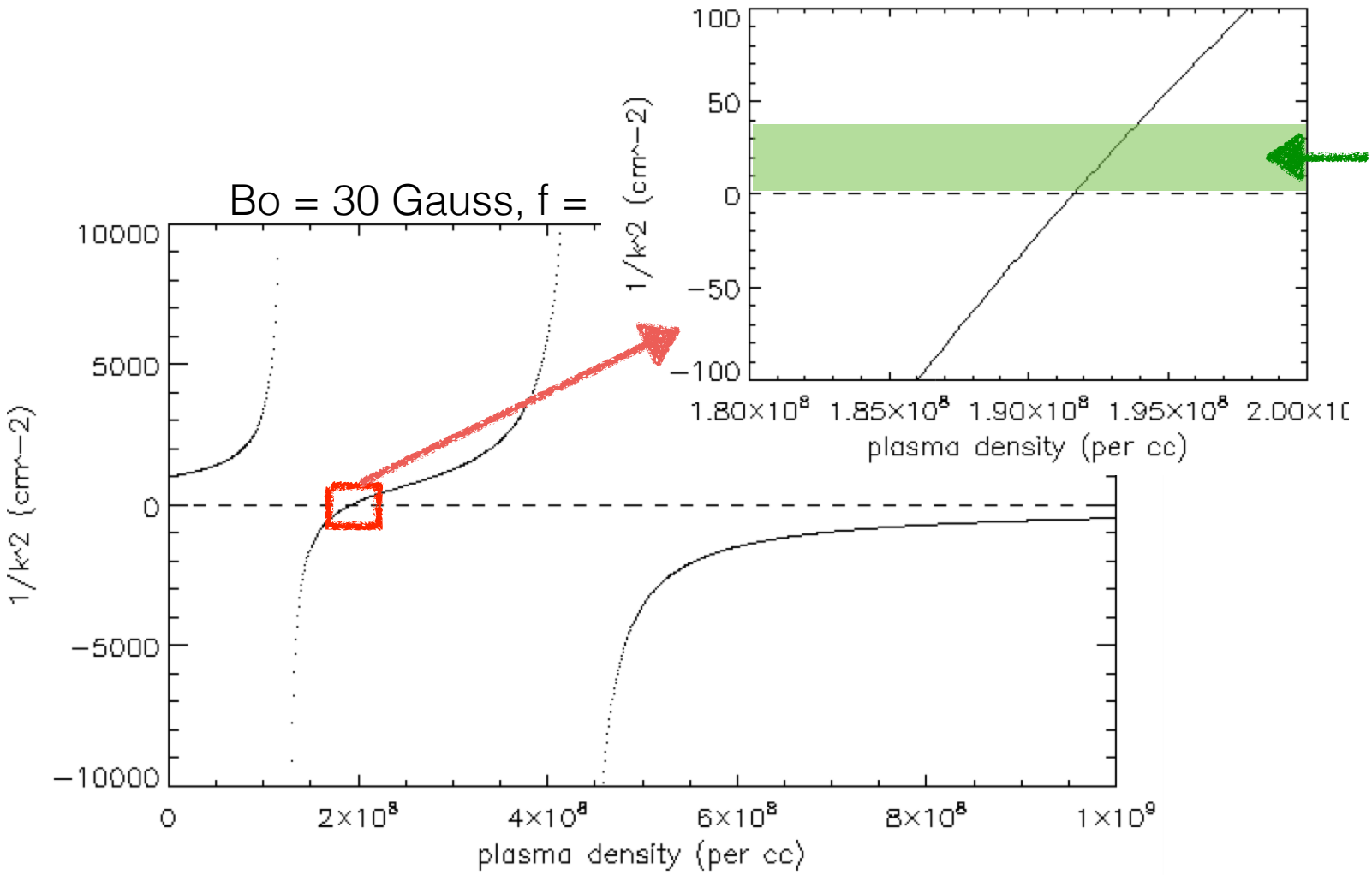
Sugai, Hideo, et al. "Mode conversion and electron heating by oblique injection of the ordinary mode into over-dense plasma." Journal of the Physical Society of Japan 57.9 (1988): 3020-3028.

X wave

- X wave dispersion relation (cold plasma): $k_\perp^2 = \frac{\omega^2}{c^2}(1 - \frac{\omega_p^2}{\omega^2}\frac{\omega^2 - \omega_p^2}{\omega^2 - \omega_{UH}^2})$



… and why it's hard to launch

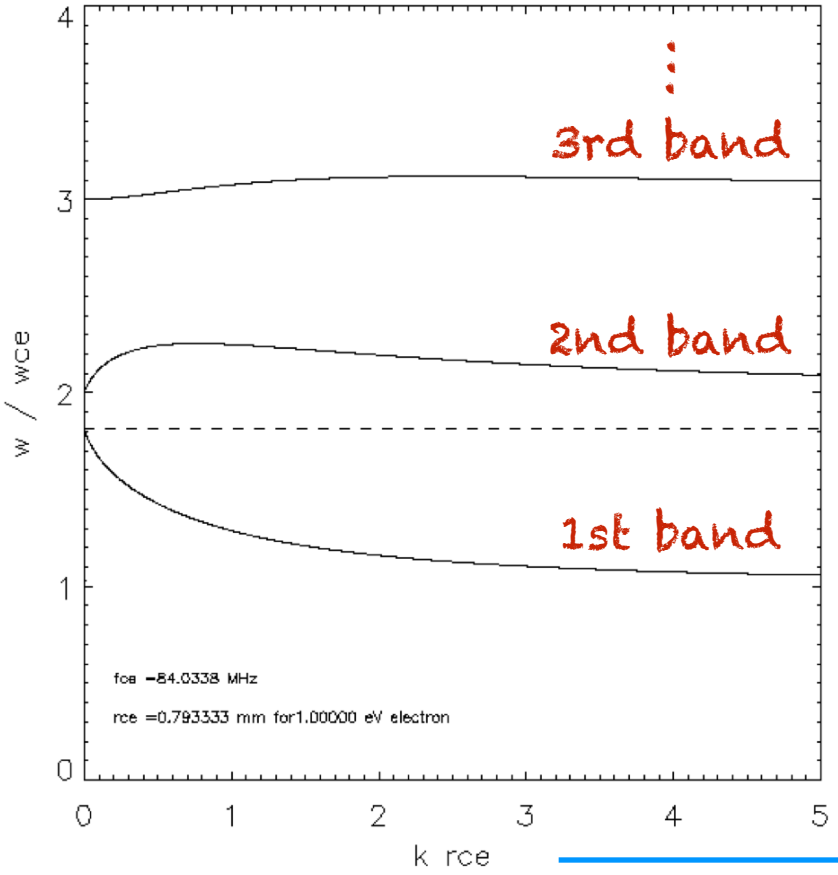


small enough to fit into the LAPTAG plasma column

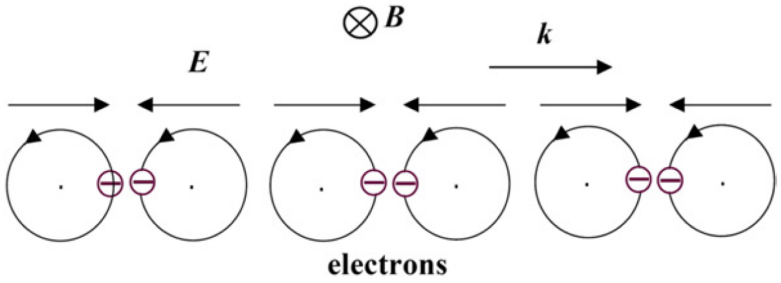
electron Bernstein wave

dispersion relation: $$k^2 = \sum_n \frac{2n^2\omega_{pe}^2\omega_{ce}^2}{\omega^2 - n^2\omega_{ce}^2}\frac{m_e}{\kappa T_e} I_n\left(\frac{k^2}{\omega_{ce}^2}\frac{\kappa T_e}{m_e}\right)\exp\left(-\frac{k^2}{\omega_{ce}^2}\frac{\kappa T_e}{m_e}\right)$$

Bo = 30 Gauss, n_e = 2e8 cm^-3



fupper hybrid



shorter wavelength

Connection between X wave and EBW

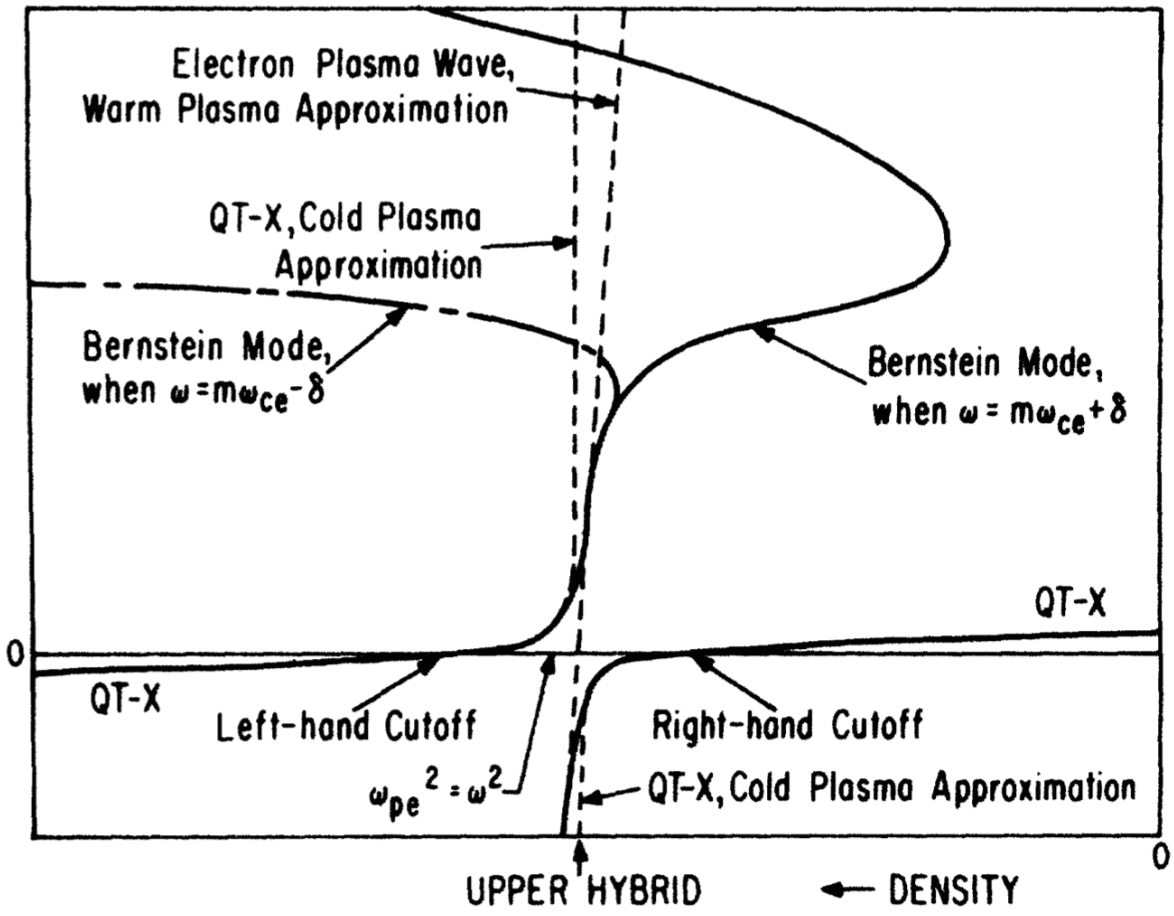
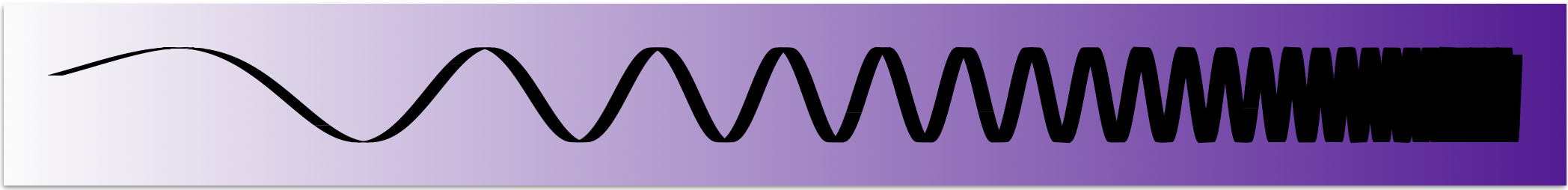


FIG. 1. The square of the index of refraction (n_x^2) is plotted against density.

Stix, Thomas H, Physical Review Letters 15.23 (1965): 878.

WKB method

Assume a wave is propagating in x direction.
The plasma is not uniform in that direction.
Then what's the wave amplitude E as function of location?



- uniform case

$$\frac{d^2E}{dx^2} + k_0^2 E = 0$$

$$E(x) = A_1 e^{ik_0 x} + A_2 e^{-ik_0 x}$$

- non-uniform case

$$\frac{d^2E}{dx^2} + k^2(x)E = 0$$

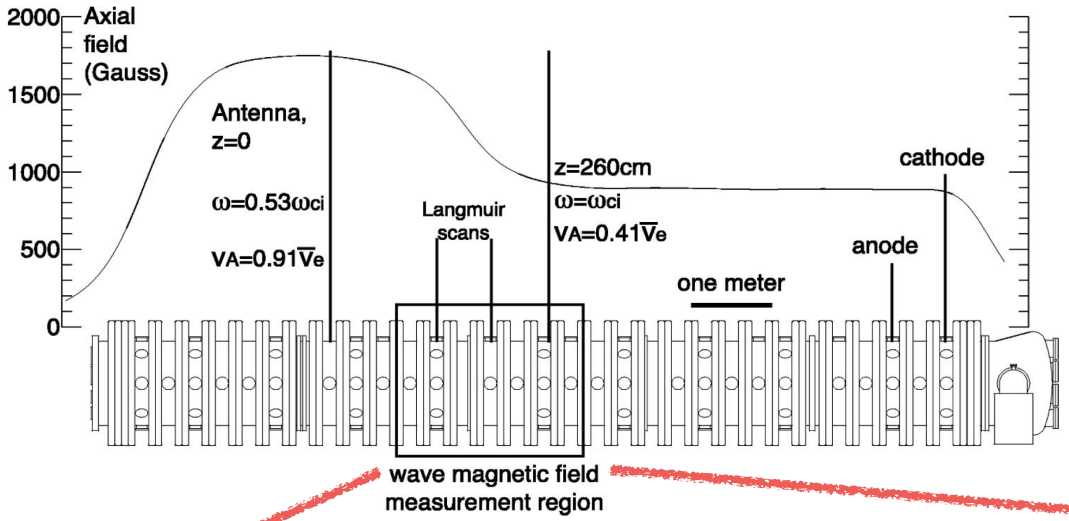
$$Assume \quad E(x) = A(x)e^{iS(x)}$$

A(x) and S'(x) varies slowly with x

$$S(x) = \pm \int^x k(x')dx'$$

$$A(x) = \frac{A_0}{\sqrt{k(x)}}$$

An example of plasma wave in 1-D inhomogeneity experiment vs WKB



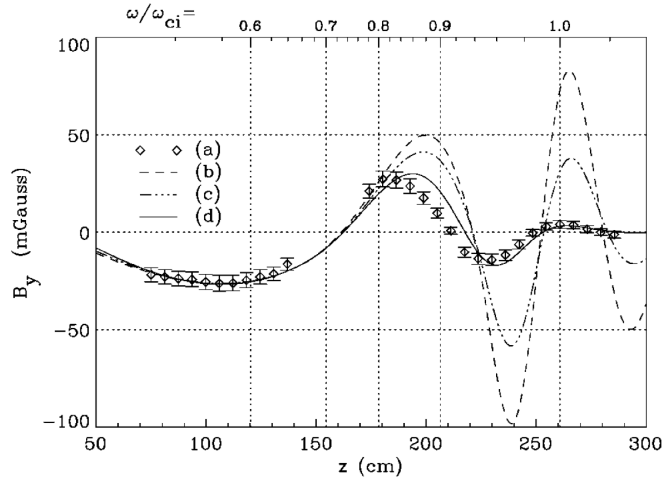
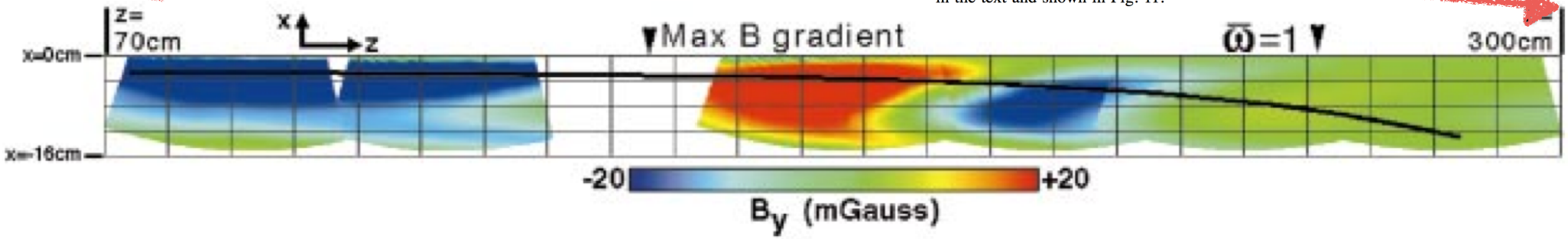
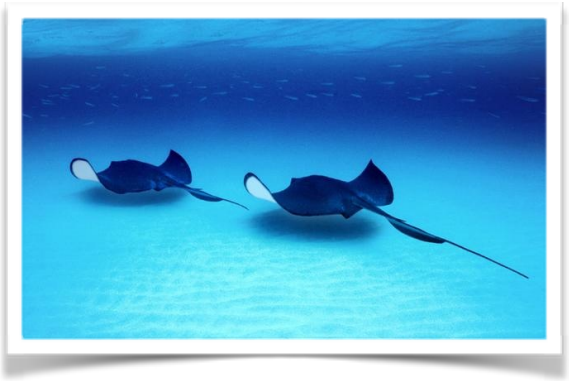


FIG. 12. A comparison of B_y axial profiles between (a) measured data points, and the WKB model of Eq. (8), with $k_\parallel$ calculated considering (b) ion-cyclotron damping only; (c) ion-cyclotron damping plus electron Landau damping; (d) the same as (c) plus electron–ion Coulomb collisions. The data are taken at time $\tau = 5.4$ and interpolated along a ray path as discussed in the text and shown in Fig. 11.



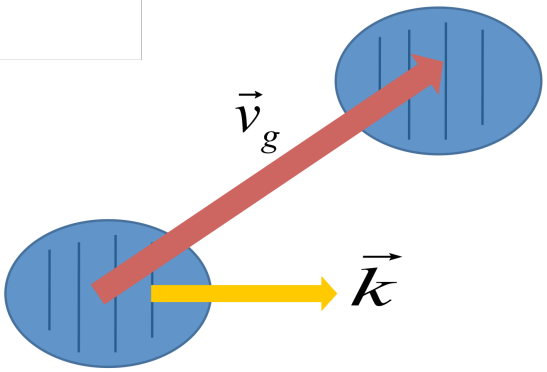
Vincena, Stephen, Walter Gekelman, and James Maggs. "Shear Alfven waves in a magnetic beach and the roles of electron and ion damping." Physics of Plasmas (1994-present) 8.9 (2001): 3884-3896.

Ray tracing

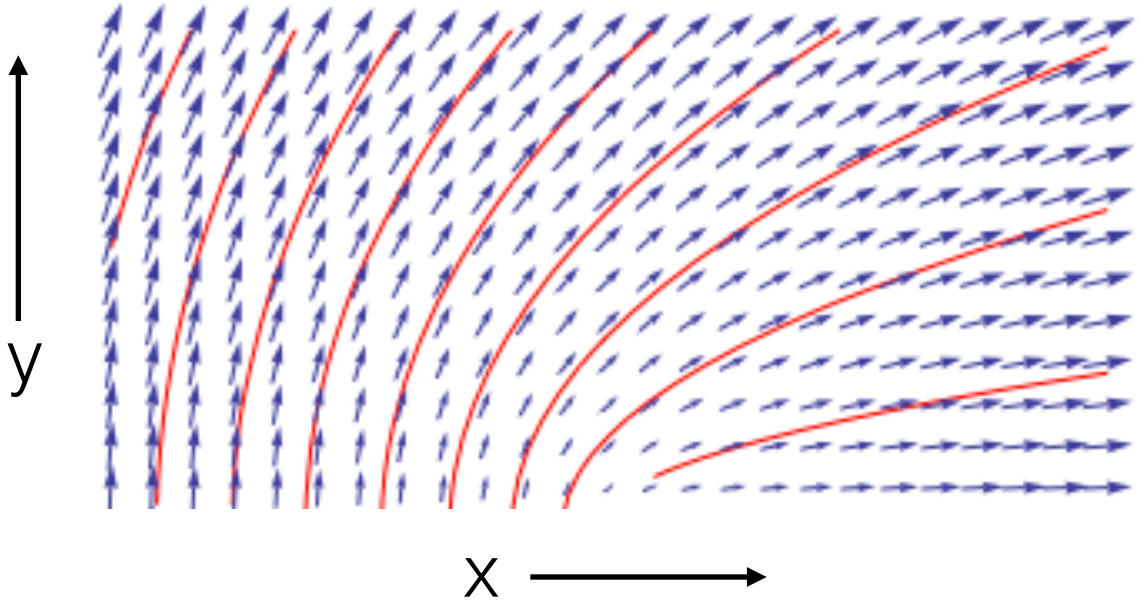


Group velocity is the velocity of a wave packet propagation

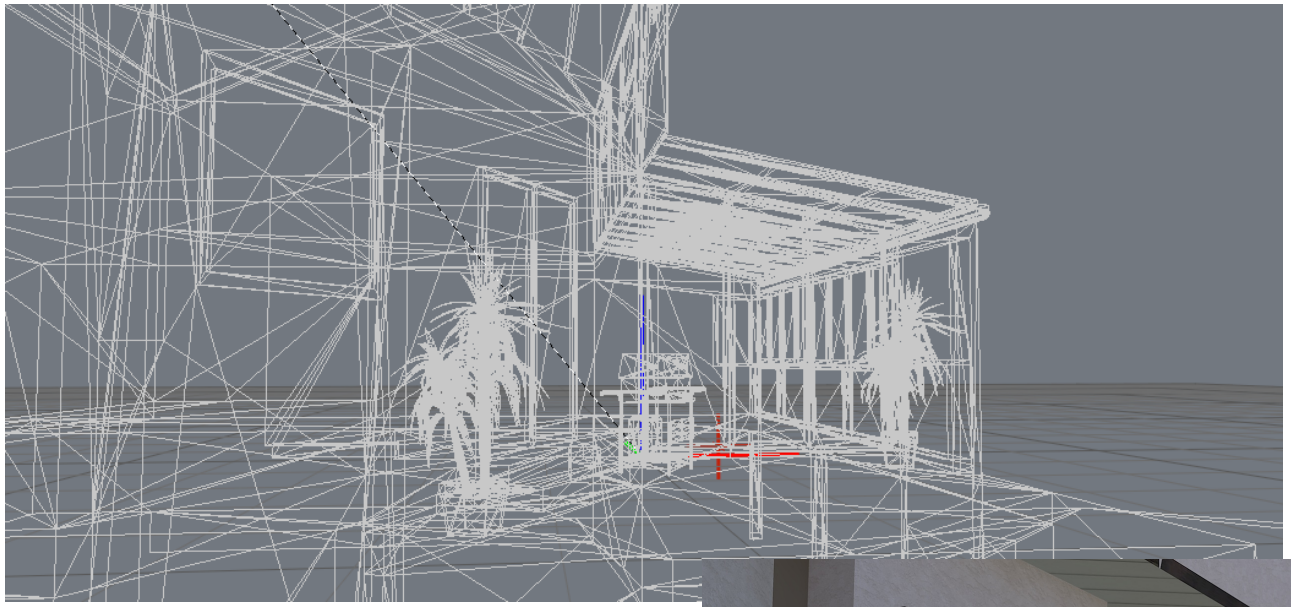
$$\vec{v}_g = \frac{\partial \omega}{\partial \vec{k}}$$



if group velocity can be calculated at any point in space, one can trace the full trajectory of a single ray



A common application of ray-tracing - computer graphics



Full wave solution

Full wave solution will take into account the influence of the merging of two different types of wave modes, along with tunneling, reflection, conversion and absorption.

$$\frac{d^4E_y}{d\xi^4} + \lambda^2\left(\xi\frac{d^2E_y}{d\xi^2} + E_y\right) = 0\,, \tag{6}$$

where

$$\xi = (\omega_c/\omega)^2 X,\quad \lambda^2 = (4\omega_c^2 - \omega^2)\omega^4 c^2/3\omega_c^6 v_t^2 L^4\,. \tag{7}$$

far away from conversion region

Asymptotic solutions (for $\lambda|\xi|^{3/2} \gg 1$ and $\lambda^2\xi^2 \gg 1$) are given by[13]

$$(-\xi)^{1/2} I_1[2(-\xi)^{1/2}] \rightarrow -\xi^{1/2} J_1(2\xi^{1/2}) \tag{8}$$

$$i2(-\xi)^{1/2} K_1[2(-\xi)^{1/2}] \rightarrow \pi\xi^{1/2} H_1^{(2)}(2\xi^{1/2}) - f_-(\xi)\,, \tag{9}$$

$$-i2(-\xi)^{1/2} K_1[2(-\xi)^{1/2}] \rightarrow \pi\xi^{1/2} H_1^{(1)}(2\xi^{1/2}) - f_+(\xi)\,, \tag{10}$$

where

$$f_\pm(\xi) = \frac{\pi^{1/2}\exp[\pm i(2\lambda\xi^{3/2}/3 - \pi/4)]}{\xi^{5/4}\lambda^{3/2}}\,. \tag{11}$$

Shoucri, Merit, and H. H. Kuehl, Physics of Fluids (1958-1988) 24.7 (1981): 1395-1396.

Status of Research

Theory	Experiment
Stix 1965	
Gorman 1966	
Kuehl et al, 1967	
Kuehl et at, 1970	
Swanson 1976	
Ngan et al, 1977	Sugai 1981
Shoucri et al, 1980	
Shoucri et al, 1981	
Ram et al, 2000	
…	