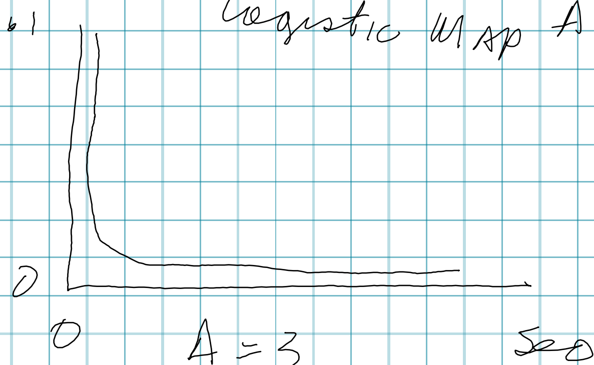
1/24/15 Walter's lecture

Logistic Map

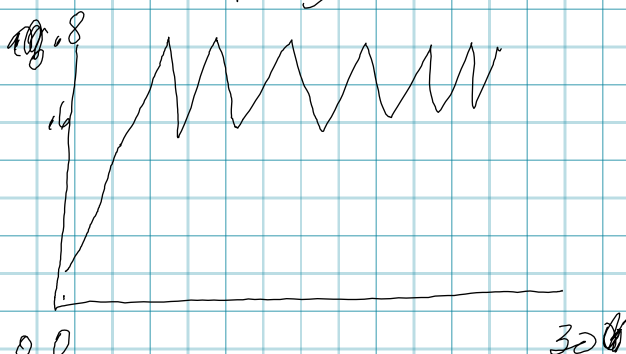
$X_{n+1} = A X_n (1 - X_n)$

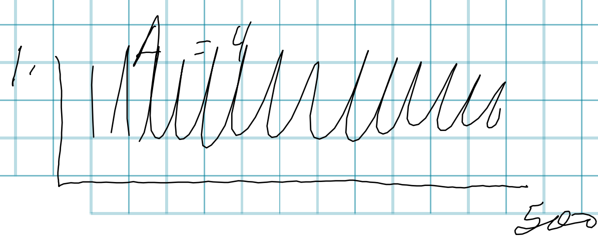
Chaotic $A = 4$

Logistic Map $A = 1$



$A = 3$



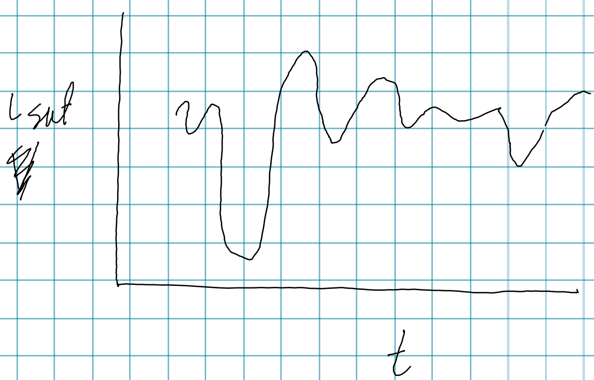


Entropy: Statistical definition

$$S = -K_B \sum_i p_i \ln p_i$$

Boltzmann (1870)'s

p_i = probability i^{th} possible microstate of the system.



$$p_j(\pi) = \frac{\#\{t_j \,/\, t_j \le T-n, (X_{t_j+1}, \ldots, X_{t+n})\}}{T-n+1}$$

has permutations type π

Probability of occurrence of π

Time series B_x has 1000 time steps

case I $n=4$ $B_x(t_0, t_1, t_2, t_3)$

$t_0 > t_2 > t_3 > t_1$

suppose case 1 happens 30 times.

$$S(P) \equiv -\sum_{j}^{N} p_j \ln(p_j)$$

$$p_1(\text{case } 1) = -\left(\frac{30}{996}\right) \cancel{\log} \ln\left(\frac{30}{996}\right)$$

~~Feat~~

Jensen Shannon Complexity.

$$C_J^S = -2 \, \frac{S\left(\frac{P+P_e}{2}\right) - \frac{1}{2}S(P) - \frac{1}{2}S(P_e)}{\frac{N+1}{N}\ln(N+1) - 2\ln(N) + \ln(N)}$$

time series N points

$S(P)$ = Shannon Entropy

$H(P)$ = Normalized Shannon Entropy

P_e = maximum entropy state,

$p_j = 1/N$

low entropy → ability to do work -

Complexity & Entropy

$$C^{K}_{V,q}[P] = Q^{V}_{q}[P]\, H^{K}_{q}[P]$$

generalized complexity

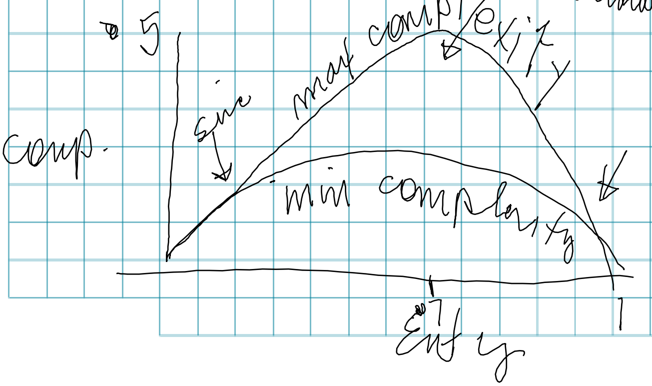
diseqilibrium

generalized entropy

K = different types of complexity

$$Q^{K=S}_{q=1} = S\left(\frac{P+P_e}{2}\right) - \frac{1}{2}S(P) - \frac{1}{2}S(P_e)$$

our data



O A Rosso et al
PRL, "Distinguishing Noise from chaos"
154102 (200_).

stochastic
deterministic

Suppose $n = 3$, $n! = 6$

(123) (132) (213) (231) (312) (321)

state 1 2 3 4 5 6

let each state represent an axis in (this case) a 6 dimension probability space

$p = [p_1\ p_2\ p_3\ p_4\ p_5\ p_6] = \{p_i\}$

$\sum_i p = 1$

min entropy system is in one state

$\{0, 0, 1, 0, 0, 0\}$ all are in $\{213\}$ state

Six possible states with min entropy

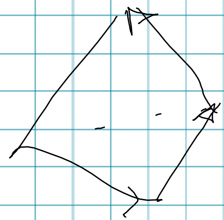
max entropy system is in all state

$p = \{ 1/6, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6} \}$

Min complexity route.
starts at maximum entropy.
centered of probability space) and
makes a beeline to a vertex
of known probability such

as $\{0, 0, 1, 0, 0, 0\}$

Eg consider $(1, 3, 2)$ prob $= \frac{1}{6}$
as it moves to a vertex
so it has $p = 1$ other 5 state
must decrease uniformly
if $P(1\,3\,2)$ becomes. $\frac{1}{2}$? all
other state now have $P = \frac{1}{10}$
beeline = shortest



max complex route stars at max. goes to min

$P_e = [\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}]$

$P_{c5,3} = [\frac{1}{5} \; \frac{1}{5} \; 0 \; \frac{1}{5} \; \frac{1}{5} \; \frac{1}{5}]$

C q H

① Min complexity states will have ~~the~~ max many states occupied

② Max complexity state will have min states occupied.

B = 160 G

Ar, 3